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# TO THE NATURE OF 0<sup>+</sup> STATES IN <sup>228</sup>Th STUDIED BY TWO-NEUTRON TRANSFER

Sequences of states observed in the <sup>230</sup>Th(p, t)<sup>228</sup>Th reaction are selected which can be treated as rotational bands and as multiplets of excitations. Moments of inertia have been derived from these sequences, whose values may be considered as evidence of the two-phonon nature of most  $0^+$  excitations. Experimental data are compared with interacting boson model and quasiparticle-phonon model calculations and with experimental data for <sup>229</sup>Pa. Conclusions have been made concerning the nature of  $0^+$  states in <sup>228</sup>Th.

*Keywords*: 0<sup>+</sup> states, collective bands, moments of inertia, nuclear models.

#### Introduction

The nucleus <sup>228</sup> Th is located in a region where strong octupole correlations are important in the properties already of the low-lying excitations. Besides the interplay of collective and singleparticle excitations, which takes place in deformed rare earth nuclei, the reflection asymmetry additionally complicates the picture of excitations. Already in an earlier publication [1], a conclusion was made that the nature of the first excited  $0^+$ states in the actinide nuclei is different from that in the rare earth region, where they are due to the quadrupole vibration. The strong excitations in the (p, t)-reaction suggest that these states represent a collective excitation different from the  $\beta$ -vibration. Decay modes of the levels of the band on the first excited  $0^+$  state in <sup>228</sup>Th have led to the suggestion that this band might predominantly have an octupole two-phonon structure [2]. One has to expect a complicated picture at higher excitations: residual interactions could mix the one-phonon and multiphonon vibrations of quadrupole and octupole character with each other and with quasiparticle excitations. Detailed experimental information on the properties of such excitations is needed. On the experimental side, two-neutron transfer reactions are very useful. On the theoretical side, a test of the advanced Interacting Boson Model (IBM) and a microscopic approach, such as the quasiparticlephonon model (QPM), would be very interesting. Spectroscopic data obtained in the foregoing paper [3] are analyzed here to throw light onto the nature of  $0^+$  states in <sup>228</sup> Th.

## Collective bands in <sup>228</sup>Th

After the assignment of spins to all excited states, those sequences of states can be identified, which show the characteristics of a rotational band structure. An identification of the states attributed to rotational bands was made on the basis of the following conditions:

a) the angular distribution for a band member candidate state is fitted by DWBA calculations for the spin necessary to put this state in the band;

b) the transfer cross section in the (p, t) reaction to states in the potential band has to decrease with increasing spin;

c) the energies of the states in the band can be fitted approximately by the expression for a rotational band  $E = E_0 + AI(I + 1)$  with a small and smooth variation of the rotational inertial parameter  $A = \hbar^2/2J$ , J is the moment of inertia (MI).

Collective bands identified in such a way are shown in Fig. 1. The procedure can be justified in that some sequences meeting the above criteria are already known from gamma-ray spectroscopy to be rotational bands [2, 4], so similar sequences are rotational bands, too. The straight lines in Fig. 1 support the argument for these assignments. It is worth to mention, that the assignments of  $0^+$  even for the states at 1531.7 and 2335.9 keV are supported by one  $2^+$  state on top of them. The bands built on states of one-phonon octupole-quadruplet (the band  $K^{\pi}$  =  $=1^{-1}$  was not correctly identified in [2]), the band with  $K^{\pi} = 0^+$ , 831.9 keV [2, 4],  $K^{\pi} = 0^+$ , 938.6, 1120.1 keV [4] and  $K^{\pi} = 2^{+}$ , 968.8 and 1153.5 keV [2, 4] were identified earlier. Additional levels are added to these bands from the (p, t) study. Two bands with  $K^{\pi} = 2^+$  are added, based only on the analysis of the decay of <sup>228</sup> Ac [2]: at 1638.23 keV and at 1899.98 keV. There is only contradiction in the spin assignment for the 2010.17 keV level. The (p, t) data do not support spin  $4^+$  and prefer a  $2^+$ assignment instead.



Fig. 1. Collective bands based on the  $0^+$ ,  $2^+$ ,  $1^-$ ,  $2^-$  and  $3^-$  excited states in <sup>228</sup>Th as assigned from the DWBA fit of the angular distributions from the (p, t) reaction.

In Fig. 2 we present moments of inertia (MI) obtained by fitting the level energies of the bands displayed in Fig. 1 by the expression  $E = E_0 + AI(I + 1)$  for close-lying levels, i.e. they were determined for band members using the ratio of  $\Delta E$  and  $\Delta[I(I + 1)]$ , thus saving the spin dependence of the MI. This procedure is valid for all bands except the 943.8 keV, 1<sup>-</sup> band. The usual procedure leads to strongly staggering values. In the case of the  $K^{\pi} = 0^{-}$  and  $K^{\pi} = 1^{-}$  bands, the Coriolis interaction mixes the band members only for odd *I*. The even members *I* of the  $K^{\pi} = 1^{-}$  band remain unperturbed. In a simple two level model ( $K^{\pi} = 0^{-}$  and  $K^{\pi} = 1^{-}$  bands) the following expression can be obtained for the band energies

$$E(I, K^{\pi} = 1^{-}) \approx E_1 + (A_1 + B)I(I + 1) \text{ for } I \text{ odd},$$
$$E(I, K^{\pi} = 1^{-}) \approx E_1 + A_1I(I + 1) \text{ for } I \text{ even, } (1)$$

where  $E_1 = E'_1 - A_1$  and  $B = C^2/(E'_1 - A_1 - E'_0)$ ,  $E'_0$ and  $E_1$  are the intrinsic bandhead energies,  $A_1$  is the inertial parameter and C is the strength of the Coriolis interaction, which is believed to be small.



Fig. 2. Moments of inertia for the bands in  $^{228}$ Th as assigned from the angular distributions from the  $^{230}$ Th(p, t) $^{228}$ Th reaction. The values of  $J/h^2$  are given in units MeV<sup>-1</sup>.

The effective parameters of inertia behaves then as

$$A_1^{\text{eff}} = A_1 + \frac{1}{2}B(I+1)$$
 for *I* odd,  
 $A_1^{\text{eff}} = A_1 - \frac{1}{2}B(I-1)$  for *I* even. (2)

Fitting these expressions to the experimental data gives the smoothly changing values of the moment of inertia between 76.9 and 78.6 MeV<sup>-1</sup>, as shown in Fig. 2 with parameter  $B = 0.75 \div 0.68$  (thus staggering is removed).

The obtained MI cover a broad range, from about 50 MeV<sup>-1</sup> to about 100 MeV<sup>-1</sup>. The negative parity bands based on the states with spin 1<sup>-</sup>, interpreted as the octupole-vibrational bands [2, 4], have high MI. The 0<sup>+</sup> band at 1120.1 keV, considered as  $\beta$ -vibrational band, has the smallest MI, close to the one of the ground-state (g.s) band. At this stage, it is difficult to assert about clear correlation between the intrinsic structure of the bands and the magnitude of their MI. Nevertheless, one can assume for the 0<sup>+</sup> bands that some of the larger MI could be related to the two-phonon octupole structure and the smallest MoI could be related to the one-phonon quadrupole structure. The bands with intermediate values of the

MoI could be based on the two-phonon quadrupole excitations. If the moments of inertia do indeed carry information on the inner structure of the bands, then the number of excitations with a structure as in the g.s. or  $\beta$ -vibrational states in <sup>228</sup>Th is small.

#### Quadruplet of octupole excitations

The lowest negative-parity excitations with  $K^{\pi} = 0^{\circ}$ , 1<sup>\circ</sup>, 2<sup>°</sup> and 3<sup>°</sup> are generally interpreted as octupole vibrational ones. They are one-phonon octupole excitations. The corresponding energies in <sup>228</sup>Th such as 328.0, 944.2, 1123.0, and 1344.0 keV were established already in [4]. Here we confirmed these assignments after the removal of some ambiguities. They are the bandheads of the rotational bands which are displayed in Fig. 4 together with the (p, t) cross sections and their parameters A.

In the case of <sup>230</sup>Th, the assumption was made that the strongly excited first  $0^+$  state, together with also strongly excited states with spins  $2^+$  and  $4^+$ , accompanied by somewhat weaker excited state with the spin  $6^+$ , belong to the two-phonon octupole quadruplet [6]. Strong excitations and close rotational parameters were the arguments for assigning the same structure to these states. As one can see in Fig. 3, the first  $0^+$  state in <sup>228</sup>Th at 831.9 keV is also strongly excited. Taking into account the decay properties of the band on this  $0^+$ state, the suggestion was made that this band has an octupole two-phonon structure [2]. The picture for other states is not so transparent. There is no prominent excitation strength of the  $2^+$  and  $4^+$  states just above this  $0^+$  state. The first excited  $2^+$  state, which is not a member of the  $0^+$  band, is the state at 968.8 keV. But the de-excitation of the band built on this state demonstrates the properties expected for a β-vibrational band. Moreover, its moment of inertia is much smaller than the one derived for the  $0^+$  band at 831.9 keV. For the band built on the  $2^+$  state at 1153.3 keV, the moment of inertia is close to the one of the band built on the  $0^+$  state at 831.9 keV and the state at 1153.3 keV is relatively strong excited in the (p, t) reaction.





Fig. 3. Experimental distribution of the (p, t) strength integrated in the angle region  $7.5^{\circ}$  -  $30^{\circ}$  for  $0^{+}$ ,  $2^{+}$  and  $4^{+}$  states in <sup>228</sup>Th.

The non-rotational  $4^+$  and  $6^+$  states, which do not belong to rotational bands, which are strongly excited in the (p, t) reaction and could be members

Fig. 4. Suggested multiplets of states of the octupole onephonon (bottom) and the octupole two-phonon (top) excitations and the corresponding collective bands.

of the two-phonon octupole quadruplet, are the states at 1643.8 and 1905.8 keV. The level at 1812.7 keV, tentatively assigned as a  $6^+$  state, can be

attributed to the band based on the  $4^+$  state, the corresponding rotational inertial parameter again is very close to the one for  $0^+$  and  $2^+$  bands. No members of a rotational band can be related to the  $6^+$  band head at energy 1905.8 keV (see Fig. 4).

#### **IBM calculations**

In phenomenological IBM, the positive-parity states are described by introducing *s* and *d* bosons, while for the negative parity states one has to introduce additional bosons with odd values of angular momentum (at least one *f* boson). In the region of transitional actinides, where octupole deformation might develop, the IBM-*spdf* (which uses *p* and *f* bosons) was applied with success in Refs. [7 - 9].

In the present paper, we adopt the IBM-spdf framework for calculating the low-lying positive and negative parity states in <sup>228</sup>Th. In Ref. [8], the IBM calculations for this nucleus have been already performed. However, these calculations used only a simplified Hamiltonian to describe the existing (up to that date) electromagnetic data. More recent calculations (which also used a simplified Hamiltonian) [10] indicated that IBM fails completely to reproduce the (p, t) spectroscopic factors. The calculated first excited states were found with a transfer strength of  $\simeq 1$  % of that of the ground state and the higher states were even weaker, whereas experimentally the first excited state is seen with  $\simeq 30$  % of the ground-state intensity. In order to treat these spectroscopic observables in a reasonable approach, we used the method suggested in Ref. [11], where it was shown that the addition of the second-order O(5) Casimir operator in the Hamiltonian can account for the observed (p, t) spectroscopic factors.

The Hamiltonian employed in the present paper is similar to the one used in Refs. [8, 9] and is able to describe simultaneously the positive and negative parity states:

$$\hat{H}_{spdf} = \varepsilon_d \hat{n}_d + \varepsilon_p \hat{n}_p + \varepsilon_f \hat{n}_f + k (\hat{Q}_{spdf} \cdot \hat{Q}_{spdf})^{(0)} + a_3 [(\hat{d}^{\dagger} \tilde{d})^{(3)} \times (\hat{d}^{\dagger} \tilde{d})^{(3)}]^{(0)}, \qquad (3)$$

where  $\mathbf{\epsilon}_{d}$ ,  $\mathbf{\epsilon}_{p}$  and  $\mathbf{\epsilon}_{f}$  are the boson energies and  $\hat{n}_{p}$ ,

 $\hat{n}_d$  and  $\hat{n}_f$  are the boson number operators, k is the quadrupole-quadrupole interaction strength and  $a_3$  is the strength of the O(5) second order Casimir operator. In the *spdf* model, the quadrupole operator is considered as [12]:

$$\hat{Q}_{spdf} = \hat{Q}_{sd} + \hat{Q}_{pf} = (\hat{s}^{\dagger}\tilde{d} + \hat{d}^{\dagger}\hat{s})^{(2)} + \chi^{(2)}_{sd}(\hat{d}^{\dagger}\tilde{d})^{(2)} + \frac{3\sqrt{7}}{5}[(p^{\dagger}\tilde{f} + f^{\dagger}\tilde{p})]^{(2)} + \chi^{(2)}_{pf}\left\{\frac{9\sqrt{3}}{10}(p^{\dagger}\tilde{p})^{(2)} + \frac{3\sqrt{42}}{10}(f^{\dagger}\tilde{f})^{(2)}\right\}.$$
 (4)

Tilde applies only to the *d*, *p* and *f* bosons and means:  $\tilde{d}_m = (-1)^m d_{-m}$ .

The quadrupole electromagnetic transition operator is:

$$\hat{T}(E2) = e_2 \hat{Q}_{spdf} , \qquad (5)$$

where  $e_2$  represents the boson effective charge. To ensure no-vanishing E2 transitions between the states containing no *pf* bosons and those having  $(pf)^2$ components we follow the approach described in Refs. [8, 9], where the mixing of different positive parity-states with different *pf* components is achieved by introducing in the Hamiltonian a dipoledipole interaction term of the form:

$$\hat{H}_{int} = \alpha \hat{D}_{spdf}^{\dagger} \cdot \hat{D}_{spdf} + h.c., \qquad (6)$$

where

$$\hat{D}_{spdf} = -2\sqrt{2} [p^{\dagger}\tilde{d} + d^{\dagger}\tilde{p}]^{(1)} + \sqrt{5} [s^{\dagger}\tilde{p} + p^{\dagger}\tilde{s}]^{(1)} + \sqrt{7} [d^{\dagger}\tilde{f} + f^{\dagger}\tilde{d}]^{(1)}$$
(7)

is the dipole operator arising from the O(4) dynamical symmetry limit, which does not conserve separately the number of positive and negative parity bosons [12, 13]. This term will also be important later in the calculations of the two-neutron transfer intensities. The interaction strength is given by the  $\alpha$  parameter and is chosen to have a very small value,  $\alpha = 0.0005$  MeV, similar to Refs. [8, 9].

For the *E*1 transitions, a linear combination of the three allowed one-body interactions was taken:

$$\hat{T}(E1) = e_1[\chi_{sp}^{(1)}(s^{\dagger}\tilde{p} + p^{\dagger}\tilde{s})^{(1)} + (p^{\dagger}\tilde{d} + d^{\dagger}\tilde{p}) + \chi_{df}^{(1)}(d^{\dagger}\tilde{f} + f^{\dagger}\tilde{d})^{(1)}], \qquad (8)$$

where  $e_1$  is the effective charge for the *E*1 transitions and  $\chi_{sp}^{(1)}$  and  $\chi_{df}^{(1)}$  are model parameters.

The goal of the present paper is to describe simultaneously both the existing electromagnetic

and the hadronic (transfer strength) data. To achieve this goal, the two-neutron transfer intensities between the ground state of the target nucleus and the excited states of the residual nucleus were also calculated. The L = 0 transfer operator may contain various terms, but we shall restrict it to the following form:

$$\hat{P}_{v}^{(0)} = (\alpha_{p}\hat{n}_{p} + \alpha_{f}\hat{n}_{f})\hat{s} + \alpha_{v}\left(\Omega_{v} - N_{v} - \frac{N_{v}}{N}\hat{n}_{d}\right)^{1/2} \left(\frac{N_{v} + 1}{N + 1}\right)^{1/2}\hat{s}, \quad (9)$$

where  $\Omega_{\nu}$  is the pair degeneracy of neutron shell,  $N_{\nu}$  is the number of neutron pairs, *N* is the total number of bosons, and  $\alpha_p$ ,  $\alpha_f$ , and  $\alpha_{\nu}$  are constant parameters. The L = 0 transfer operator contains two additional terms beside the leading order term, proportional to the bosonic  $\hat{s}$  operator [14]. Details about the contributions of different terms in calculating the (p, t) spectroscopic factors will be given in a forthcoming paper [15].

The calculations were performed using the computer code "OCTUPOLE" [16]. The Hamiltonian is diagonalized in a Hilbert space with a total number of bosons  $N_B = n_s + n_d + n_p + n_f$ . For the present calculations we used an extended basis allowing up to three negative parity bosons  $(n_p + n_f = 3)$ . The vibrational strengths used in the calculations are  $\varepsilon_d = 0.2$  MeV,  $\varepsilon_p = 1.0$  MeV, and  $\varepsilon_f = 1.1$  MeV, while the quadrupole-quadrupole interaction strength has a value of k = -21 keV. The strength of the O(5) second order Casimir operator is set to  $a_3 = 0.053$  MeV, while the quadrupole operator parameters are  $\chi_{sd}^{(2)} = -1.09$  and  $\chi_{pf}^{(2)} = -1$ .

The full spectrum of excited  $0^+$  states obtained in the present experiment is displayed in Fig. 5 in comparison to the corresponding calculated values. In the energy range covered experimentally (up to 2.5 MeV), in the IBM-spdf calculations one obtains 10 excited  $0^+$  states in comparison to the 17 experimentally observed  $0^+$  excitations (firm spin assignment). Given that there was no attempt to fine tune the calculations to the empirical  $0^+$  states, there is no point in invoking a precise energy cut-off for the IBM calculations. Therefore, it is appropriate to look also above 2.5 MeV, where there is a continuous spectrum of  $0^+$  states consisting of 20 states up to 3.3 MeV and as many as 30 up to 4 MeV. The IBM asserts that some of these states are having 2 pf bosons in their structure and are related (according to Ref. [8]) to the presence of double dipole/octupole excitations. For example, the boson admixtures for the first excited  $0^+$  state are  $n_d = 4.2$ ,  $n_p = 1.4$  and  $n_f = 0.6$  in comparison with those for the  $\beta$ -vibrational state as  $n_d = 4.5$ ,  $n_p = 0.006$  and  $n_f = 0.001$ . However, the present theoretical and experimental data cannot allow for a final decision on the nature of the  $0^+$  states. In particular, additional experimental information is needed to measure the branching ratios and the absolute transition probabilities stemming from these states. In Fig. 5, the  $2^+$  and  $4^+$  levels revealed in the present experiment are also compared to the results of the IBM calculations. The experiment revealed 33 excited  $2^+$  and 25 excited  $4^+$  states up to 2.5 MeV. In the same energy range, the calculations produced only 16 excited  $2^+$  states and 15 excited  $4^+$  states. If one looks above this limit, one finds 32 excited  $2^+$  states and 33 excited  $4^+$  states up to 3.3 MeV within the IBM.

In nucleus <sup>228</sup>Th there are no lifetimes measured for the negative-parity states, hence no absolute transition probabilities might be extracted. Therefore, we would restrict the present discussion to reproducing the B(E1)/B(E2) ratios. A detailed comparison between the experimental data and the present calculations is presented in Table 1. The best agreement is obtained by using  $e_1 = 0.005$  efm and  $e_2 = 0.19$  eb as the effective charges in Eqs. (8) and (5), respectively. The remaining *E*1 parameters are  $\chi_{sp} = 0.4$  and  $\chi_{df} = -1.4$ .

The B(E1)/B(E2) ratios discussed in Table 1 belong to the  $K^{\pi} = 0^+_1$  (the predicted doubleoctupole phonon band) and  $K^{\pi} = 0^+_3$  ( $\beta$ -vibrational) bands. The comparison between them is important, because it can be used as a tool for providing additional information about the nature of the  $K^{\pi} = 0^{+}_{1}$  band. All the states belonging to this band are having 2 pf bosons in their structure in the IBM calculations and are supposed to have a doubleoctupole phonon character. Further information confirming this hypothesis comes from the analysis of the E1 and E2 branching ratios. By using the IBM picture, the states belonging to the  $K^{\pi} = 0^+_1$  band will show the strong transitions into the negativeparity states (if they have a double-octupole character), while the levels stemming from the  $K^{\pi} = 0_3^+$  ( $\beta$ -band) will show relatively weak E1 transitions to these states. The experimental values in Table 1 fully confirm this hypothesis, showing that the B(E1)/B(E2) ratios are at least one order of magnitude larger for the  $K^{\pi} = 0^{+}_{1}$  band.

In Fig. 6, we display the calculated two-neutron intensities for  $^{228}$ Th as compared to the integrated experimental cross sections normalized to that of the ground state. The calculations reproduce the strong excitation of the first 0<sup>+</sup> state at 832 keV in roughly good agreement with the experiment. The experimental spectrum of 0<sup>+</sup> states is dominated also by a single state located at an energy of 2.1 MeV, showing a high cross section of about 15 % of that of the ground state. In the IBM, there is a state located at 2.1 MeV, which have the transfer intensi-



Fig. 5. Energies of all experimentally assigned excited  $0^+$ ,  $2^+$ , and  $4^+$  states in <sup>228</sup>Th in comparison to *IBM-spdf* calculations. For the  $0^+$  excitations, the states containing 2 *pf* bosons in their structure and assumed to have a double dipole/octupole character are marked with asterisk.

Κ <sup>π</sup>	E <sub>i</sub> , keV	$I_i$	I <sub>fl</sub>	I <sub>f2</sub>	Exp. 10 <sup>-4</sup> b <sup>-1</sup>	IBM 10 <sup>-4</sup> b <sup>-1</sup>	
$0_{1}^{+}$	832	$0^+$	$1_{1}^{-}$	$2_{1}^{+}$	5.1(4)	6.1	
	874	$2^{+}$	$3_1^-$	<b>4</b> <sup>+</sup> <sub>1</sub>	7.1(15)	7.6	
		$2^{+}$	$3_1^-$	$2_{1}^{+}$	24.5(31)	15.2	
		$2^{+}$	$3_1^-$	$0_{1}^{+}$	14.7(24)	23.6	
		$2^{+}$	$1_{1}^{-}$	$4_{1}^{+}$	4.2(9)	4.4	
		$2^{+}$	$1_{1}^{-}$	$2_{1}^{+}$	14.5(19)	8.9	
		$2^{+}$	$1_{1}^{-}$	$0_{1}^{+}$	8.7(14)	13.7	
	968	4 +	$5_{1}^{-}$	6 <sub>1</sub> +	22.8(80)	9.2	
		4 +	$5_{1}^{-}$	<b>4</b> <sup>+</sup> <sub>1</sub>	10.8(27)	18.2	
		4 +	$5_{1}^{-}$	$2_{1}^{+}$	6.7(13)	20.7	
		4 +	$3_1^-$	6 <sub>1</sub> +	19.1(67)	5.9	
		4 +	$3_1^-$	4 <sup>+</sup> <sub>1</sub>	9.0(23)	11.8	
		4 +	$3_1^-$	$2_{1}^{+}$	5.6(11)	13.4	
03+	1176	$2^{+}$	$1_{1}^{-}$	4 <sup>+</sup> <sub>1</sub>	0.060(25)	0.08	
		$2^{+}$	$1_{1}^{-}$	$2_{1}^{+}$	0.25(10)	0.27	
		$2^{+}$	$1_{1}^{-}$	$0_{1}^{+}$	0.62(28)	0.38	
		$2^+$	$3_1^-$	<b>4</b> <sup>+</sup> <sub>1</sub>	0.09(5)	0.16	
		$2^{+}$	$3_1^-$	$2_{1}^{+}$	0.39(20)	0.52	
		$2^{+}$	$3_1^-$ .	$0_{1}^{+}$	0.95(51)	0.72	

Table 1. Experimental and calculated B(E1)/B(E2) transition ratios in <sup>228</sup>Th. The parameters of the E1 operator are fitted to the available experimental data

ties of about 18 %. This state has a double-octupole phonon structure. Another state at 2.29 MeV with a relative cross section of about 7 % can be put in correspondence to the calculated in the IBM state at 2.2 MeV also with a double-octupole phonon structure. The running sum in Fig. 6 is taken up to 3.25 MeV, where another group of states with significant transfer strength is calculated by the IBM. The parameters from Eq. (9) were estimated from the fit of the known two-neutron transfer intensities (integrated cross sections). The values employed in the present paper are  $\alpha_p = 1.3$  mb/sr,  $\alpha_f = -0.4$  mb/sr, and  $\alpha_v = 0.03$  mb/sr. The location and transfer intensity of the strongest states is very well reproduced by the calculations. Because the calculated energy distribution of the 0<sup>+</sup> states is





Fig. 6. Comparison between the experimental two-neutron transfer strengths (panel (*a*)) for the  $0^+$  states and the IBM results (panel (*b*)). In panel (*c*) the experimental versus computed running sum of the (p, t) strengths is given. (See color Figure online.)

underestimating the experimental data, this also affects the fragmentation of the transferred strength. However, the main characteristics are well reproduced by the present calculations.

#### **QPM** calculations

The IBM is a phenomenological approach. To gain a detailed information on the properties of the states excited in the (p, t) reaction, a microscopic approach is necessary (see, g.e., QPM, self-consistent RPA on the basis of HFB). The ability of the QPM to describe multiple  $0^+$  states (energies, *E*2 and *E*0 strengths, two-nucleon spectroscopic factors) was demonstrated for <sup>158</sup>Gd [17]. An extension of the QPM to describe the  $0^+$  states in the actinides [18] was made after our publication on the results of a preliminary analysis of the experimental data [10]. These calculations are used for comparison with the present detailed analysis of the experimental data for <sup>228</sup>Th. See [18, 19] for the theoretical basis of the calculations.

The experimental spectrum of the  $0^+$  relative reaction strength for the (p, t) transfer (the ratios of the (p, t) strength for every state to those for the ground state) is compared to the results of the QPM calculations in Fig. 7. The (p, t) normalized transfer

Fig. 7. Comparison of the experimental (*a*) and calculated with the QPM (*b*)  $0^+$  relative reaction strengths for the (p, t) reaction. The values for the  $0^+_{gs}$  are normalized to 1. The experimental increments of the (p, t) strength (absolute values) in comparison to the QPM calculations are shown in the panel (*c*). (See color Figure online.)

spectroscopic strengths in the QPM are expressed also as ratios

$$S_n(p,t) = \left[\frac{\Gamma_n(p,t)}{\Gamma_0(p,t)}\right]^2.$$
 (10)

The amplitude  $\Gamma_0(\mathbf{p}, t)$  refers to the transitions to the I members of the ground-state rotational band, i.e. to the ground state at an analysis of the  $0^+$ excitations. The amplitude  $\Gamma_n(p, t)$  includes the transitions between the ground state and the oneand two-phonon components of the wave function. The numerical results of the calculations obtained within the QPM approach [18] are provided to us by A. V. Sushkov [20]. The QPM generates 15 states  $0^+$ below 2.5 MeV, in fair agreement with the 17 firmly assigned states. The calculations reproduce the strong excitation of the first  $0^+$  state in accordance with the experiment. In Fig. 7, we present also the increments of the (p, t) strength ratios in comparison to those of the QPM calculated normalized spectroscopic strengths. As one can see, the calculations of a few first peaks are roughly in agreement with the experiment. The (p, t) strength for the questionable  $0^+$  state at 2335.9 keV does not influence considerably the results of comparison (hence it is not included in the comparison).

A visible deviation of the calculated strength from experiment is seen above 2 MeV. Theory predicts many  $0^+$  excitations at higher energies (more than 80 states in the energy range below 4 MeV), but with a small strength. At the same time, two strong excitations are observed in the experiment at 2.13 and 2.29 MeV (Table 2), respectively. It is interesting to note that both the IBM and the QPM describe the two strongly excited states and therefore a jump in the increments of the (p, t) strength in the vicinity of 2 MeV, thus reproducing partly the sharp increase in the experimental increment.

Table 2. Phonon structure of selected QPM states. The weights of the one-phonon  $(|(\lambda \mu)|_i)$  or the two-phonon  $(|(\lambda \mu)_i((\lambda \mu)_i)|)$  components are given in percent. Only main one-phonon components are shown. Transfer factors  $S(\mathbf{p}, \mathbf{t})$  are normalized to the  $0^+_{g.s.}$  state value

$K_n^{\pi}$	$E_n^{exp}$	$E_n^{\ calc^*}$	$E_n^{\ calc^{**}}$	$S_n^{exp}$ SS	$S_n^{\ calc^{**}}$	Structure from [18]	Structure from [4]
01+	0.832	0.8	0.724	0.236	0.281	(20)196	$(20)_197; [(30)_1(30)_2]0.3$
$0_{2}^{+}$	0.939	1.0	1.496	0.041	0.002	$(20)_294;(20)_34$	$(20)_295; [(30)_1(30)_1]0.7$
$0_{3}^{+}$	1.120	1.2	1.570	0.008	0.001	$(20)_24;(20)_393$	$(20)_3 82; (20)_4 14; (20)_2 1$
$0_{4}^{+}$	1.511	1.4	1.831	0.013	0.021	$(20)_455; (20)_58; (20)_612;$ $[(30)_1(30)_1]21$	$(20)_478; (20)_317$
$0_{5}^{+}$	1.532			0.003			
$0_{6}^{+}$	1.628	1.6	1.950	0.058	0.089	$(20)_4 20; (20)_5 73; (20)_6 3$	$(20)_597; [(30)_1(30)_1]0.5$
$0_{7}^{+}$	1.691	1.7	1.962	0.008	0.046	$(20)_415; (20)_512; (20)_662;$ $[(30)_1(30)_1]6$	$(20)_692; [(30)^1(30)_1]4$
$0_{8}^{+}$	1.710	1.8	2.138	0.003	0.001	$(20)_7 89; [(22)_1 (22)_1] 4$	$(20)_797; [(30)_1(30)_2]0.2$
$0_{9}^{+}$	1.750			0.011			
$0_{10}^{+}$	1.909	1.9	2.162	0.028	0.016	$(20)_69; (20)_853;$ $[(22)_1(22)_1]4; [(30)_1(30)_1]23$	$(20)_874; (20)_94; (20)_64$
$0_{11}^{+}$	2.045		2.190	0.003	0.008	$(20)_{7}6; [(22)_{1}(22)_{1}]87$	
$0_{12}^{+}$	2.080		2.270	0.028	0.013	$(20)_43; (20)_65; (20)_837$ $(20)_914; [(30)_1(30)_2]32$	
$0_{13}^{+}$	2.131		2.290	0.150	0.008	$(20)_85; (20)_980;$ $[(30)_1(30)_1]6; [(31)_1(31)_1]7$	
$0_{14}^{+}$	2.159		2.334	0.007	0.001	$(20)_94; [(30)_1(30)_2]87$	
$0_{15}^{+}$	2.290		2.350	0.067	0.003	$(20)_{10}2; (20)_{14}30;$ $[(22)_1(22)_2]65$	
$0^{+}_{17}$	2.456		2.359	0.003	0.001	$(20)_{10}84;$ [ $(30)_1(30)_1$ ]3; [ $(30)_1(30)_3$ ]3	
$0_{1}^{-}$	0.328	0.5		0.005			(30) <sub>1</sub> 99
$1_{1}^{-}$	0.944	1.0		0.002			(31) <sub>1</sub> 98
$3_1^-$	1.344	1.4		0.002			$(33)_195; [(20)_1(33)_1]3$
$2_{1}^{+}$	0.969	1.0		0.121			$(22)_1 98$
$2_{2}^{+}$	1.153	1.3		0.145	17		$(22)_2 99$
$4_{1}^{+}$	1.432	1.5		0.001			(44) <sub>1</sub> 99

\* Data are taken from [4].

\*\* Data are taken from [18]

In these QPM calculations, the dominant phonon structure of the  $0^+$  states in the low part of energies is of the one-phonon quadrupole nature. For higher energies, an admixture of the two quadrupole and two octupole phonons are present in the structure of these states, and for some of the states they become dominant. The relatively modest role of the octupole

phonons in the structure of the low energy  $0^+$  states is explained in [18] by the enhancement of the Pauli principle, leading to a spreading of the lowest two octupole phonon components among several QPM  $0^+$  states and pushing them to higher energies.

Besides the publication [18], other calculations of the same Dubna group were carried out for a semimicroscopic description of the level structure, and transition rates between excited states in <sup>228</sup>Th, observed in the decay of <sup>228</sup>Pa [4]. The wave functions, the level energies from these two publications in correspondence to the experimental ones and to the transfer factors are given in Table 2 too. The transfer factors and also the moments of account to put in inertia are taken into correspondence the experimental and calculated levels. The large difference in the transfer factors is seen only for two levels at 2131 and 2290 keV. There is an essential difference in the energies of the lowest  $0^+$  states in [4, 18]. They are more close to the experimental ones in [4]. This is caused by the choice of the isoscalar quadrupole-quadrupole interaction strength stronger than the critical value in Ref. [4]. As a consequence the energy of the lowest collective  $0^+$  state becomes imaginary and its properties such as the structure, E2 reduced transition probabilities and the transfer factor are partially transferred to the next  $0^+$  collective state. Among the transition properties, the calculated B(E1)/B(E2) ratios at the decay of some  $0^+$  states are of special interest in [4].

## Table 3. Experimental and calculated B(E1)/B(E2) transition ratios in <sup>228</sup>Th. The Weisskopf estimate of this ratio is B(E1)/B(E2) = 2.9 b<sup>-1</sup>

$K_i^{\pi}$	E <sub>i</sub>	I <sub>i</sub>	E <sub>fl</sub>	I <sub>f1</sub>	E <sub>f2</sub>	I <sub><i>f</i>2</sub>	Exp., 10 <sup>-4</sup> b <sup>-1</sup>	QPM, 10 <sup>-4</sup> b <sup>-1</sup>
$0^+$	831.8	$0^+$	328.0	1-	57.8	$2^{+}$	5.1(4)	2.25
$0^+$	938.6	$0^+$	328.0	1-	57.8	$2^{+}$	6.7(6)	54.8
$0^+$	1175.5	$2^{+}$	328.0	1-	57.8	$2^{+}$	0.06(3)	1.45

In Table 3 we present some data on the B(E1)/B(E2) ratios, in order to note the difference in the explanation of the experimental data by the QPM and the IBM. The calculated ratios for transitions from the  $0_1^+$ ,  $0_2^+$ , and  $0_3^+$  states to the octupole  $0_1^$ state and the ground state band are compared with the experimental ratios. As one can see from Tables 2 and 3, the small admixtures of the octupole twophonon components to the wave functions of the  $0_1^+$ and  $0_2^+$  states are responsible for the fast E1 transitions. In the IBM, the similar results are obtained for the  $0_1^+$  state, having 2 pf bosons in their structure and which are supposed to have mainly a double-octupole phonon character (see Table 1 and corresponding discussion). At the same time, the B(E1)/B(E2) ratio is considerably smaller for the decay of the  $0^+$  state, which is a  $\beta$ -vibrational state, again in agreement with the experiment. The same result is obtained in the IBM.

Generally, the QPM is quite accurate in nuclei with small ground-state correlations. These correlations increase with the collectivity of the first one-phonon states, which is exactly the case of <sup>228</sup>Th. To illustrate it we performed the simple QPM calculations using the Nilsson potential + monopole pairing interaction (Nilsson parameters k and  $\mu$  taken from Ref. [21], deformation parameters  $\varepsilon_2 = 0.167$ ,  $\varepsilon_4 = -0.080$  and pairing gaps  $\Delta_n = 0.732$  MeV,  $\Delta_p =$ = 0.901 MeV taken from Refs. [22, 23]) + isoscalar quadrupole-quadrupole and octupole-octupole interaction. In the calculations only one-phonon RPA states were taken into account. From Fig. 8, one can see that the contribution  $S_n^{\phi}(\mathbf{p}, \mathbf{t})$  of the "backward" RPA amplitudes to the normalized relative transfer-reaction spectroscopic strength  $S_{n}(\mathbf{p},\mathbf{t})$  is important for the first excited  $0^{+}$  onephonon state indicating thus its pairing vibrational character. The maximum value of the number of quasiparticles with the quantum number q in the ground state,  $n_{\text{max}}^{20}$ , can be calculated (see [4])

$$n_{\max}^{20} = \max\left[\frac{1}{2}(\varphi_{qq}^{20})^2\right],$$
 (11)

where  $\phi_{qq}^{20}$  are the "backward" RPA amplitudes of the first  $0^+$  one-phonon state. For the isoscalar quadrupole-quadrupole interaction strength  $k_{20}$  = = 0.55 keV fm<sup>-4</sup>, that reproduces the experimental energy of the first  $0^+$  excited state, the ground state correlations measured by  $n_{\text{max}}^{20}$  become large (see Fig. 8). The effect of multi-phonon admixtures that two-octupole phonon poles, pushes and consequently, the two-octupole phonon energies to lower values is then underestimated. In summary, the accuracy of the calculations of <sup>228</sup>Th as stated in Ref. [4] is worse due to the increased ground-state correlations and the shift of two-phonon poles towards smaller energies. In a future QPM study one also has to take into account the spin-quadrupole interaction that is known to increase the density of the low-lying  $0^+$  states [21, 24].

### To the nature of 0<sup>+</sup> excitations

At a microscopic approach there can be a few situations of structure of the  $0^+$  states. A  $\beta$ -vibrational mode can be characterized by the relatively small two-nucleon transfer strength and a relatively large B(E2) value with a moment of inertia close to the one of the ground state. The large ratio B(E1)/B(E2) and the increase of the moment of



Fig. 8. The QPM energy *E* of the first excited  $0^+$  one-phonon state, the normalized relative transfer – reaction strength  $S_n(\mathbf{p}, \mathbf{t})$ , the contribution  $S_n^{\varphi}(\mathbf{p}, \mathbf{t})$  of the "backward" RPA amplitudes  $\varphi$  to  $S_n(\mathbf{p}, \mathbf{t})$  and the maximum number of quasiparticles with quantum number *q* in the ground state  $(n^{20})_{\text{max}}$  as functions of the isoscalar quadrupole-quadrupole interaction strength  $k_{20}$ . For more details see the text.

inertia indicate the presence of the octupole twophonon component. If a state has a relatively weak B(E2) value and also a weak two-nucleon transfer strength, but exhibits an increase of the moment of inertia, it should be a state with one dominant 2qp configuration. The pairing vibrational excitations can be characterized by their large two-nucleon transfer strengths and relatively small B(E2) values.

It is clear that the first and second  $0^+$  excited states cannot be the  $\beta$ -vibrational states, as usually is observed in deformed rare-earth nuclei since their moments of inertia are much larger than those of the ground state and also their (p, t) strengths are large. The actual  $\beta$ -vibrational state is observed at 1120 keV and it is excited very weakly in the (p, t) reaction. As we have seen, both the IBM and the QPM reproduce the  $0^+$  relative reaction strength for the (p, t) reaction and the B(E1)/B(E2) ratios of the decay of the lowest  $0^+$  states reasonably well. At the same time, the nature of  $0^+$  excitations in the QPM differs significantly from the one in the *spdf-IBM*. In all low-lying states of the QPM calculations, the quadrupole phonons are dominant and the octupole phonons are predicted to play a relatively modest role, whereas the IBM shows the lowest  $0^+$  state as having mainly 2pf bosons in their structure [8]. The analysis of the lowest quadrupole phonon wave function in the QPM reveals that the backward RPA amplitudes  $\varphi$  contribute considerably to the relative (p, t) reaction strength indicating thus that the lowest excited  $0^+$  state describes pairing vibrations arising from ground state fluctuations [18].

For an additional hint to the nature of the lowest  $0^+$  states, we include Fig. 9 with the B(E1)/B(E2) ratio stemming from the lowest excited states in <sup>228</sup>Th and <sup>230</sup>Th. Intuitively, one would expect that a large B(E1)/B(E2) ratio might be characteristic for a two-octupole-phonon excitation, whereas a small ratio might indicate a shape oscillation. Such a picture is observed for <sup>228</sup>Th: The large B(E1)/B(E2) ratios for the  $0^+_1$  and  $0^+_2$  states, and vanishing values for the  $\beta$ - and  $\gamma$ -vibrational states.

In this aspect a comparison of the relative transfer strengths in the (p, t) reaction leading to the  $0^+$  states in <sup>228</sup>Th and to the  $3/2^-$  states in <sup>229</sup>Pa (data are taken from [25]) has to be considered in addition (Fig. 10). Since the ground-state spin in the target nucleus  $^{231}$  Pa is  $3/2^{-}$ , just these spins are excited in a two-neutron L = 0 transfer. The nucleus <sup>229</sup>Pa can be regarded as <sup>228</sup>Th plus a strongly coupled proton. The rotational band built on the first  $3/2^{-1}$  excited state in <sup>229</sup>Pa at the energy 11.6 keV [26] corresponds to the <sup>228</sup>Th g.s. band and is excited very strongly. The main component in the structure of the ground state in the target nucleus <sup>231</sup>Pa is 1/2[530] [25]. Therefore, the levels excited in the L = 0transfer are  $I^{\pi}$ ,  $K = 3/2^{-}$ , 1/2 states, and they are members of the collective bands based on the state originating from coupling the  $K^{\pi} = 1/2^{-1}$  state to the <sup>228</sup>Th core-excited states. Such bands, as identified in [24], can be used to derive the moments of inertia for at least three  $3/2^{-1}$  states. The values of  $J/\hbar^{2}$  in MeV<sup>-1</sup> are given below (energies in <sup>229</sup>Pa are relative to the lowest  $3/2^{-}$  state)

<sup>229</sup> Pa:	E (keV)	0.0	703	830	1524
	$J/\hbar^2$ , MeV <sup>-1</sup>	79.2	78.4	127.5	89.3
<sup>228</sup> Th:	E (keV)	0.0	832	939	
	$J/\hbar^2$ , MeV <sup>-1</sup>	51.9	70.3	73.2	

The moments of inertia in <sup>229</sup>Pa are larger than those in <sup>228</sup>Th, that is a manifestation of the contribution of the odd proton. The large moment of inertia for the 830.5 keV state can probably be explained (at least partly) by neglecting the Coriolis coupling when fitting the energies of the corresponding band. Nevertheless, the increment of the moment of inertia for the state at 830.5 keV



Fig. 9. Comparison of the B(E1)/B(E2) values for the lowest excited  $0^+$  levels in  $^{228}$ Th and  $^{230}$ Th.

From the incomplete information, we can only conclude that the 831.8 keV state in <sup>228</sup>Th has the largest pairing vibrational branch and in <sup>229</sup>Pa the additional proton has the effect that the largest pairing vibrational branch is moved to the 1523.7 keV state. No theoretical explanation was undertaken since the publication of the experimental results on <sup>229</sup>Pa [25]. It would be interesting to undertake the theoretical analysis of the excitations with the  $L = 0^+$  transfer in odd nuclei, and first of all in the <sup>229</sup>Pa nucleus. According to the experimental study of excitations in other odd nuclei, as we have seen, different phenomena can be expected.

As for the experimental evidence of the nature of other  $0^+$  states, we have only the moments of inertia derived from the sequences of states treated as rotational bands, and thus, an only tentative conclusion can be drawn about their structure. In

relative to other states in <sup>229</sup>Pa can be put in correspondence to similar increments for the states at 831.8 and 938.6 keV relative to the g.s. in <sup>228</sup>Th. But this state, as well as other low-lying states in <sup>229</sup>Pa, is only weakly excited in contrast to the strong excitation of the state 831.8 keV in <sup>228</sup>Th. At the same time, the  $3/2^-$  state with the energy at 1523.7 keV is excited strongly, but it cannot be put in correspondence to the first excited state in <sup>228</sup>Th: There is practically no increment of its moment of inertia relative to the lowest  $3/2^-$  state. Besides that, its energy is almost twice larger than the first excited  $0^+$  state in <sup>228</sup>Th.



Fig. 10. The (p, t) cross sections for the  $0^+$  states in <sup>228</sup>Th and  $3/2^-$  states in <sup>229</sup>Pa divided by the cross sections for the ground state in <sup>228</sup>Th (*a*) and for the lowest  $3/2^-$  state in <sup>229</sup>Pa (*b*), in percent. Energies in <sup>229</sup>Pa are given relative to the lowest  $3/2^-$  state.

contrast to <sup>230</sup>Th [6], for which they are distributed almost uniformly over the region from 47 to 98 MeV<sup>-1</sup>, the moments of inertia in <sup>228</sup>Th have values close to 50 MeV<sup>-1</sup> only for the g.s.,  $\beta$ -vibrational 0<sup>+</sup> states and for the state at 1531.7 keV, all other 0<sup>+</sup> states have larger values from 70 to 95 MeV<sup>-1</sup>. This fact can indicate that corresponding states are probably of a two-phonon nature too, or two quasi-particle states with an admixture of the pairing vibrations.

#### Conclusion

The firm assignments for the  $0^+$ ,  $2^+$  and  $4^+$  and some  $6^+$  states in <sup>228</sup>Th allowed to suggest multiplets of states, which can be treated as the one- and twophonon octupole quadruplets, and to identify the sequences of states, which have the features of rotational bands with definite inertial parameters. Moments of inertia are derived from these sequences. Only for the g.s. and  $\beta$ -vibrational states and additionally for the state at 1531.7 keV, the moments of inertia are about 50 MeV<sup>-1</sup>. For all other states they are larger than 70 MeV<sup>-1</sup>, i.e. the value for the first excited 0<sup>+</sup> state. This information, together with the spectroscopic information on some  $\gamma$ -transitions, were used for conclusions on the nature of the 0<sup>+</sup> states. The experimental data have been compared to the IBM-*spdf* and QPM calculations. Spectroscopic factors from the (p, t) reaction, and the trend in their change with excitation energy, are approximately reproduced by

both the IBM and QPM for the  $0^+$  states. A remarkable feature of the IBM and QPM is the existence of the strong first vibrational excitations, close in magnitude and position to the experimental ones. Giving also an approximately correct number of  $0^+$  states, these models provide different descriptions for the structure of these states. The lack of additional information does not allow for final conclusions on the validity of the theoretical approaches. Challenging experiments on gamma spectroscopy following (p, t) reactions would give much needed information.

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# ДО ПРИРОДИ 0<sup>+</sup> СТАНІВ У <sup>228</sup>Th, ЩО ЗБУДЖУЮТЬСЯ ПРИ ДВОНЕЙТРОННІЙ ПЕРЕДАЧІ

Вибрано послідовності збуджених станів, що спостерігались у  $^{230}$ Th(p, t) $^{228}$ Th реакції, які можуть розглядатись як ротаційні смуги та мультиплети збуджених станів. Моменти інерції були отримані з цих послідовностей, величини яких можуть розглядатись як свідоцтво двофононної природи більшості 0<sup>+</sup> збуджень. Експериментальні дані порівнюються з розрахунками в рамках моделі взаємодіючих бозонів та квазічастинково-фононної моделі, а також з експериментальними даними для  $^{229}$ Pa. Обговорюється природа 0<sup>+</sup> збуджень в ядрі  $^{228}$ Th.

*Ключові слова:* 0<sup>+</sup> стани, колективні смуги, моменти інерції, ядерні моделі.

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## К ПРИРОДЕ 0<sup>+</sup> СОСТОЯНИЙ В <sup>228</sup>Th, ВОЗБУЖДАЕМЫХ ПРИ ДВУХНЕЙТРОННОЙ ПЕРЕДАЧЕ

Выбраны последовательности возбужденных состояний, наблюдаемых в  $^{230}$ Th(p, t) $^{228}$ Th реакции, которые могут рассматриваться как ротационные полосы и мултиплеты возбужденных состояний. Моменты инерции были получены из этих последовательностей, величины которых могут рассматриваться как свидетельство двухфононной природы большинства 0<sup>+</sup> возбуждений. Экспериментальные данные сравниваются с расчетами в рамках модели взаимодействующих бозонов и квазичастично-фононной модели, а также с экспериментальными данными для  $^{229}$ Pa. Обсуждается природа 0<sup>+</sup> возбуждений в ядре  $^{228}$ Th.

*Ключевые слова:* 0<sup>+</sup> состояния, коллективные полосы, моменты инерции, ядерные модели.

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