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ENERGY LEVELS OF NUCLEI ^{40}Sc AND ^{40}K AS A FUNCTION OF SEMI-CLASSICAL COUPLING ANGLE $\theta_{1,2}$ WITHIN THE MODIFIED SURFACE DELTA-INTERACTION

In this work, nuclear shell model was applied using modified surface delta-interaction to calculate, in particle-hole state, the energy levels of isobar nuclei ^{40}Sc and ^{40}K. Particles are in the model space ($1f_{7/2}$) while the holes are found in the model space ($1d_{3/2}$, $1s_{1/2}$, $1d_{5/2}$). The total angular momentum and parity are identified for possible particles and holes in nuclei above. Thus, we have used a theoretical study to find relationship between energy levels and the semi-classical coupling angle $\theta_{1,2}$ at different orbitals within particle-hole configuration. We notice the energy levels seem to follow two universal functions which depend on the semi-classical coupling angles $\theta_{1,2}$. We found the theoretical data agree to the experimental data.

Keywords: shell model, energy levels, modified surface delta-interaction, ^{40}Sc, ^{40}K, particle-hole.

1. Introduction

Having simple shell model in mind, Talmi [1] used the surface delta-interaction (*SDI*) to evaluate properties of nuclear states with few 'nucleons' on a magic core. Likewise, the shell model has been successful in describing configurations with a few nucleons outside closed shells or missing from them [2]. It employs assumption: at first there exists an inert core model of close shell [3], which acts with central forces upon valence nucleons; second, there exists a residual interaction caused by two-body forces acting between the valence nucleons. Schiffer [4] proposed very suggestive method. He consider only those nuclei where two particles (2*p*), two holes (2*h*) or particle and hole are present in addition to closed shell, and move in the orbits j_1 and j_2 of a self consistent field. He pointed out the universal behavior of the effective interaction in terms of the angle between the angular momenta of the interacting nucleons, property which was later shown to be related to the short-range character of the effective interaction [5, 6].The angle $\theta_{1,2}$ between the proton (hole or particle) and neutron (hole or particle) angular momentum vectors j_1 and j_2 were reported in Refs. [7, 8]. There are a number of theoretical works discussing nuclear shell model, particle-hole (*ph*) configuration by means modified surface delta-interaction (*MSDI*). For example, shell model calculation of A = 41 - 43 nuclei [9]. The modified schematic shell model for *ph* configuration within *SDI* [1, 10] and *MSDI* has been applied to various nuclear structure states [11, 14] Lawson and Talmi [15, 16] studied the spectra ^{38}Cl, ^{40}K and ^{40}Ca by employing the Pandya transformation between particle-particle (*pp*) spectra such as ^{38}Cl and *ph* spectra as such as ^{40}K. Johnstone [17] reported calculations for the one-hole states in potassium isotopes $40 \le A \le 46$ using a model space based on the $(f_{7/2}p_{3/2})^n(d_{3/2}s_{1/2})^{-1}$ configuration. The matrix elements of the residual interaction are treated as free parameters which are determined by a least-squares fit to the observed energy levels; good agreement is achieved in his results. In Ref. [18] residual interaction theory was applied to the study of the relation between *pp* and *ph* spectra using model space $(1f_{7/2}1p_{3/2})$ for ^{40}K. The result of this study was the exposition of the Pandya *pp-ph* relation. There are some noticeable differences among the 5 sets of proton and neutron matrix elements, although the 2^- and 5^- energies appear stable. The lowest *ph* multiplet, $(1f_{7/2})(1d_{3/2})^{-1}$, is well known [19] in ^{40}Sc, ^{40}K and ^{40}Ca. N. Schulz et al. [20] described an experimental analysis for the ^{40}Ca(^{3}He, t)^{40}Sc reaction. Many experimental and theoretical studies on the structure of energy level in particle- hole states had been investigated [21, 22]. *ph* configuration is discussed in other book with the idea to use simple forces [23, 26]. Recently we studied the energy level of ^{210}Po within the framework of the *SDI* [27]. Energy levels of isobaric nuclei ^{16}N, ^{16}F within the *MSDI* were studied [28] by D. N. Hameed and A. K. Hasan. Previous studies motivate the aim of the present work by application of *MSDI* to calculate the energy levels of isobaric nuclei (^{40}Sc and ^{40}K) in *ph* state. We have used a theoretical study to find relationship between energy levels and the semi-classical coupling angle $\theta_{1,2}$ at different orbital within *ph* configuration.

2. *ph* formalism

The Hamiltonian can be written in the form [29]

$$H = \sum_{\alpha\beta} \left\langle \alpha \mid E \middle| \beta \right\rangle \eta_\alpha^+ \eta_\beta + $$

$$+ \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \left\langle \alpha\beta \mid V \middle| \gamma\delta \right\rangle \eta_\alpha^+ \eta_\beta^+ \eta_\delta \eta_\gamma \ , \qquad (1)$$

where η_β^+ and η_β denote the (creation and annihilation) operators respectively, of a nucleon in state $|\alpha\rangle$; E is the single-particle (*s. p.*) energy, and $\left\langle \alpha\beta \mid V \middle| \gamma\delta \right\rangle$ represents the normalized and antisymmetrized matrix element of nucleon -nucleon interaction. The wave function in one-particle (one-hole) (1*p* or 1*h*) configuration space are represented by [30]

$$\left|0; ph^{-1}\right\rangle = a_{hole} a^+_{particle} |0\rangle. \qquad (2)$$

The accord to denote particle state by the label p_1, p_2, p_3, p_4 … and hole state by label h_1, h_2, h_3, h_4 …,

$$H_{s.p.} \left|0; ph^{-1}\right\rangle = \left(e_{\text{particle}} - e_{\text{hole}}\right) a_{\text{hole}} a^+_{\text{particle}} |0\rangle =$$

$$= \left(e_{\text{particle}} - e_{\text{hole}}\right) \left|0; ph^{-1}\right\rangle. \qquad (3)$$

Eq. (3) is the *s. p* H, e_{particle} representing *s. p.* energy for 'particle' and e_{hole} symbolizing *s. p.* energy for 'hole' the angular momentum coupling and, once a sufficiently large set of basis function is obtained, the coupled *ph* state are given by

$$0; ph^{-1}; \left| JM \right\rangle =$$

$$= \sum_{m_{\text{particle}} m_{\text{hole}}} \left\langle j_p m_p j_h m_h \middle| JM \right\rangle \tilde{a} j_h m_h \, a^+ j_p m_p |0\rangle. \quad (4)$$

Thus one obtains for the matrix elements of the residual interaction (suppressing the symbol 0 for the close shell state)

$$\left\langle j_{p_1} j_{h_1}^{-1} ; JM \middle| V \middle| j_{p_2} j_{h2}^{-1} ; JM \right\rangle = \frac{1}{4} \sum_{\alpha\beta\gamma\delta} \left\langle \alpha\beta \middle| V \middle| \gamma\delta \right\rangle \times \sum_{m_{p1} m_{h_1} m_{p_2} m_{h_2}} (-1)^{j_{h_1} - m_{h_1}} \left\langle j_{p_1} m_{p_1} j_{h_1} - m_{h1} \middle| JM \right\rangle,$$

$$(-1)^{j_{h_2} - m_{h_2}} \times \left\langle j_{p_2} m_{p_2} j_{h_2} - m_{h2} \middle| JM \right\rangle \left\langle 0 \middle| a_{j\, p_1} m_{p_1} a^+{}_{jh1} m_{h1} N(a^+_\alpha a^+_\beta a_\delta a_\gamma) a_{jh2} m_{h2} a^+{}_{j\, p_2} m_{p_2} \middle| 0 \right\rangle. \qquad (5)$$

The ground state is taken to be a closed configuration and the excited state are mixed (1*p*, 1*h*) configuration this approach is usually referred to as the Tamm - Danceff approximation.

3. The *SDI*

Moszkowski and co-workers [31] have proposed a simple model able to describe the interaction between the valence nucleons. This model of interaction assumes the residual interaction $V_{1,2}$ between particles 1 and 2 takes place at the nuclear surface only [27, 32], interaction defined in this way possesses all the features of pairing interaction. It is short-ranging and allows only symmetric spatial states. It should be pointed out that the *SDI* exists not only between particles coupled to *T* = 1, *J* = 0 but also between particles coupled to *J* ≠ 0. Unlike pairing, *SDI* interaction acts also in states with *T* = 0. Therefore the *SDI* should be a better approximation then a pure pairing one. Following the work of Glaudcmans [33] where the isospin dependence of the interaction was taken into account the 'interaction' may be written in the form [29, 32]

$$V_{1,2} = -4\pi A_T \delta \ \Omega_{1,2} \ \delta(\hat{r}(1)) -$$

$$-R_0) \delta(\hat{r}(2) - R_0) + B \ \tau_1 \cdot \tau_2, \qquad (6)$$

where $\hat{r}(1)$ (1), $\hat{r}(2)$ are the position vectors of interacting particles, R_0 is the nuclear radius [22, 34] the strength of interaction A_T .The correction term $B \ \tau_1 \cdot \tau_2$ is introduced to account for the splitting between the groups of levels with different isospin. Such a form of interaction is called MSDI. The antisymmetrized matrix element of $V_{1,2}$ is given by [11, 12, 28, 29]

$$\left\langle j_1 j_2 \middle| V_{1,2} \middle| j_3 j_4 \right\rangle = -\frac{A_T}{2(2J+1)} \cdot \left\{ \frac{(2j_1+1)(2j_2+1)(2j_3+1)(2j_4+1)}{2(2J+1)(1+\delta_{1,2})} \right\}^{1/2} \times$$

$$\times \left[(-1)^{l_1+l_2+j_3+j_4} \left\langle j_2 - 1/2 \, j_1 1/2 \middle| J0 \right\rangle \cdot \left\langle j_4 - 1/2 \, j_3 1/2 \middle| J0 \right\rangle \cdot \left[1 - (-1)^{l_3+l_4+J+T} \right] - \right.$$

$$-\left\langle j_2 - 1/2\, j_1 1/2 \middle| J0 \right\rangle \cdot \left\langle j_4 - 1/2\, j_3 1/2 \middle| J0 \right\rangle \Big| \left[1 + (-1)^T\right] + \left[2T(T+1) - 3\right] \cdot B\ \ \delta_{1,3}\delta_{2,4}, \tag{7}$$

where $\langle j_2 - 1/2\, j_1 1/2 | J0\rangle$ and $\langle j_4 - 1/2\, j_3 1/2 | J0\rangle$ are the Clebsh - Gordon coefficients [23, 29, 32] and j_1, j_2, j_3 and j_4 are the spin states of particles. Correspondingly, J and T indicate the spin and isospin of a 2p state. The diagonal matrix elements with $j_1 = j_2$ and $j_3 = j_4$ correspond to pure states. The behavior of 2h nuclei is very much the same that 2p nuclei except that the *s. p.* energy $e_{h(\text{hole})} = -e_{p(\text{particle})}$. The residual interaction is given by the *MSDI* defined in Eq. (6), the matrix of the Hamiltonian is [28, 29]

$$\left\langle p_1 h_1^{-1} \middle| H \middle| p_2 h_2^{-1} \right\rangle_\Gamma = \left(e_{p_1} - e_{h_1}\right)\delta_{p_1 p_2}\delta_{h_1 h_2} + \left\langle p_1 h_1^{-1} \middle| V \middle| p_2 h_2^{-1} \right\rangle_\Gamma, \tag{8}$$

$$\left\langle p_1 h_1^{-1} \middle| V \middle| p_2 h_2^{-1} \right\rangle_\Gamma = \frac{1}{4}\left\{(2j_{p_1}+1)(2j_{p_2}+1)(2j_{h_1}+1)(2j_{h_2}+1)\right\}^{0.5}(-1)^Q\left[A_0\left\{1+2(-1)^{l_{h2}+l_{p2}+J}\right\} + A_1\left\{1+2(-1)^T\right\}\right]\times$$
$$\times\left\langle j_{p1} - 1/2\, j_{h2} 1/2 \middle| J0\right\rangle \cdot \left\langle j_{p2} - 1/2\, j_{h2} 1/2 \middle| J0\right\rangle + (-1)^Z\left[A_0 - A_1\left\{1+2(-1)^T\right\}\right]\cdot\left\langle j_{h1} - 1/2\, j_{p1} 1/2 \middle| J0\right\rangle\times$$
$$\times\left\langle j_{p2} - 1/2\, j_{1h2} 1/2 \middle| J0\right\rangle\ \Big]\delta^{even}_{l_{p1}+l_{p2}l_{h1}+l_{h2}} - \left[C + B\left\{1+2(-1)^T\right\}\right]\delta_{P_1 P_2}\delta_{h_1 h_2}, \tag{9}$$

where $\Gamma = TJ$, $Q = j_{h1} + j_{h2} + l_{p1} + l_{h1} + J + T$, and $Z = j_{h1} + j_{p2} + l_{p2} + l_{h1}$, J is the total angular momentum, and values of A_0, A_1, B and C as functions of the mass number A [11, 29] are obtained from fits to experimental data in the varied mass region, where $A_0 \simeq A_1 \simeq B \simeq C \simeq 25/A$. The behaviour of the diagonal two-body matrix element as a function of the spin J of 2p state is very characteristic when their value are plotted in a particular way. Consider two particles in orbits j_1 and j_2 with $J = j_1 + j_2$. One can write then [29 - 35]

$$J^2 = (j_1 + j_2)^2 = j_1^2 + j_2^2 + 2(j_1 j_2)\cos\theta_{1,2}, \tag{10}$$

where $\theta_{1,2}$ is the angle between the vectors j_1 and j_2. Since the length of vector j is given by $\sqrt{j(j+1)}$ one obtains from Eq. (10) in a semi-classical picture [34]

$$\cos\ \theta_{1,2} = \frac{J(J+1) - j_1(j_1+1) - j_2(j_2+1)}{2\sqrt{j_1(j_1+1)\,j_2(j_2+1)}}. \tag{11}$$

The J-dependence of the matrix element $\langle j_1 j_2 | V_{1,2} | j_3 j_4\rangle$ can thus be plotted as a function of the angle $\theta_{1,2}$. The radial overlaps of the particle orbits for light nuclei differ from those for heavy nuclei. The ensuing mass dependence can be removed [8] by dividing each matrix element by the absolute value of the average two-body energy. For 2-particles in the same orbit j this quantity E is defined by

$$\bar{E} = \left|\frac{\sum_{JT}(2J+1)\left\langle j^2 \middle| V \middle| j^2\right\rangle_{JT}}{\sum_J (2J+1)}\right|, \tag{12}$$

where J and T are summed over all possible values for the two-particle states $\left|j^2\right\rangle_{JT}$. The matrix element extracted from the experimental data as indication above and divided by the absolute value of the energy, $\bar{E}(j^2)$, are plotted as a function of $\theta_{1,2}$ for two particles in the same orbit. The two-body matrix element for two active particle in different orbits outside a $T_z = 0$ core are obtained from nuclei assumed to be described by a proton in one specific orbit and a neutron in another. The proton-neutron configurations correspond to nucleon pair having mixed isospin and one finds [29]

$$E_J(p,n) = =1/2\left\{\left\langle j_1 j_2 \middle| V_{1,2} \middle| j_3 j_4\right\rangle_{J,T=1} + \left\langle j_1 j_2 \middle| V_{1,2} \middle| j_3 j_4\right\rangle_{J,T=0}\right\}. \tag{13}$$

Plotting the excitation energy of these states as a function of the corresponding angle $\theta_{1,2}$ determined according Eq. (11). For proton and neutron in different orbits the absolute value of average two body energy is given by

$$\bar{E} = \left|\frac{\sum_J (2J+1)E_j}{\sum_J (2J+1)}\right|. \tag{14}$$

With E_J defined by Eq. (13). The shape of the curve presented in Fig. 1, *b*, *c* and Fig. 2, *b*, *c* for $\theta_{1,2}$ = 180. The orbits of the two interaction particles, moving in opposite direction, have a large overlap. Since the nuclear force is of short range the interaction will be strong (large and attractive). For

$\theta_{1,2}$ = 90, the particle orbits have a small overlap, which result in a weak interaction. This interaction makes clearly why the curve in Fig. 1, *b*, *c* and Fig. 2, *b*, *c* have appositive slope for $\theta_{1,2}$ varying from 180 to 90. For smaller angle $\theta_{1,2}$ the Pauli exclusion principle become important. For $\theta_{1,2}$ = 0 and $j_1 = j_2$, one must distinguish the two possibilities of isospin coupling. In the former case the particle occupy a spatially symmetric *ph* state which due to the strong short rang attraction lead to a large negative matrix element. In the case *T* = 1 case the *ph* form a spatially antisymmetric state and hence their relative distance increases for decreasing angle $\theta_{1,2}$ to 0.

4. Results and discussion

Nuclear properties of many nuclei are compared with the experimental data [36]. The energy levels of ^{40}Sc and ^{40}K nucleus are calculated according to the following details.

4.1. ^{40}Sc nucleus

In the frame of the shell model, we have considered ^{40}Sc nucleus. In this case, there are one proton (particle) and one neutron (hole) outside of the inert core ^{40}Ca. One proton occupies the model space $1f_{7/2}$, and one neutron occupies the model space $1d_{3/2}$ $2s_{1/2}$ $1d_{5/2}$. Energy levels can be obtained by using the *s. p.* energy [36, 37], where: $e_{P(particle)} = e1f_{7/2} = -1.085$ MeV, $e_{H(hole)} = e1d_{5/2} = -15.64$ MeV, $e2s_{1/2} = -13.17$ MeV and $e1d_{3/2} = -10.995$ MeV. The parameters A_0 = 0.7, A_1 = 0.8, B = 0.65 and C = –0.3. Table 1 and Fig. 1, *a* show comparison between theoretical and experiment values of ^{40}Sc nucleus [38]. The *MSDI* calculations of the energies and parity are in good agreement with the experimental values.

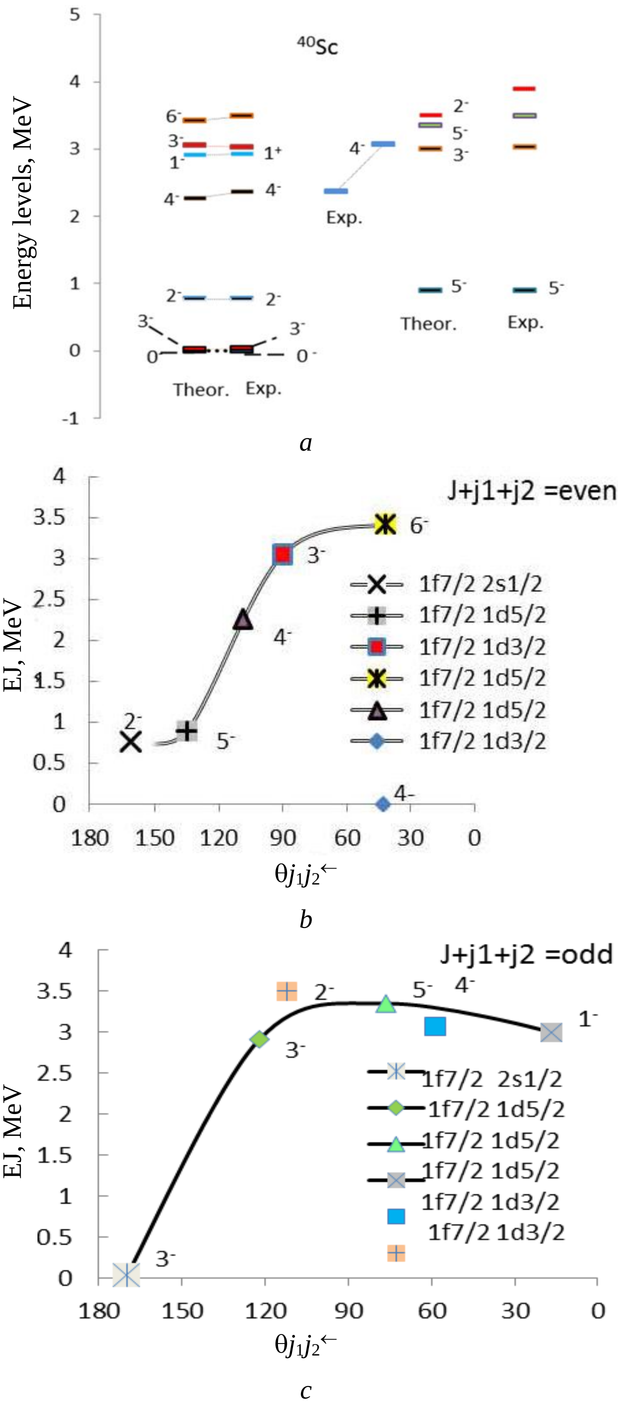


Fig. 1.

Table 1. **Matrix element values N_{Sc} and N_K (MeV) for nuclei ^{40} Sc and ^{40} K, respectively** $N = \left\langle j_{p1} j_{h1^{-1}} \middle| V \middle| j_{p2} j_{h2^{-1}} \right\rangle$

j_{p1} ^{40}Sc ^{40}K	j_{h1}^{-1} ^{40}Sc ^{40}K	j_{p2} ^{40}Sc ^{40}K	j_{h2}^{-1} ^{40}Sc ^{40}K	$\bar{J}$	N_{Sc}	N_K	j_{p1} ^{40}Sc ^{40}K	j_{h1}^{-1} ^{40}Sc ^{40}K	j_{p2} ^{40}Sc ^{40}K	j_{h2}^{-1} ^{40}Sc ^{40}K	$\bar{J}$	N_{Sc}	N_K
7/2	3/2	7/2	3/2	2	–1.843	–5.536	7/2	5/2	7/2	5/2	1	–7.707	–0.922
7/2	3/2	7/2	3/2	3	–2.351	–5.357	7/2	5/2	7/2	5/2	2	–9.212	–2.212
7/2	3/2	7/2	3/2	4	–3.073	–4.799	7/2	5/2	7/2	5/2	3	–9.953	–2.983
7/2	3/2	7/2	3/2	5	–1.993	–5.018	7/2	5/2	7/2	5/2	4	–9.983	–3.013

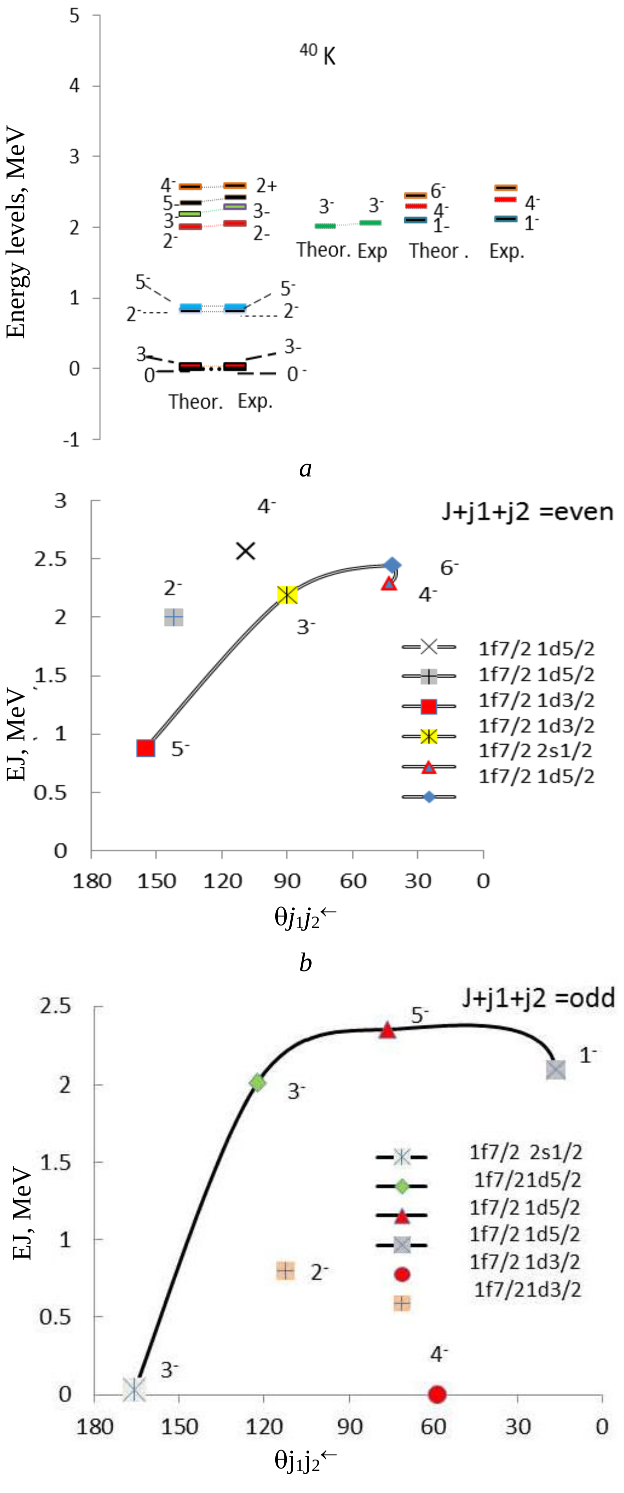
Continuation of the Table 1

j_{p1} ^{40}Sc ^{40}K	j_{h1}^{-1} ^{40}Sc ^{40}K	j_{p2} ^{40}Sc ^{40}K	j_{h2}^{-1} ^{40}Sc ^{40}K	$\bar{J}$	N_{Sc}	N_K	j_{p1} ^{40}Sc ^{40}K	j_{h1}^{-1} ^{40}Sc ^{40}K	j_{p2} ^{40}Sc ^{40}K	j_{h2}^{-1} ^{40}Sc ^{40}K	$\bar{J}$	N_{Sc}	N_K
7/2	1/2	7/2	1/2	3	–4.463	–2.694	7/2	5/2	7/2	5/2	5	–9.093	–2.132
7/2	1/2	7/2	1/2	4	–4.443	–2.587	7/2	5/2	7/2	5/2	6	–6.563	–0.566

4.2. ^{40}K nucleus

The model space includes $1f_{7/2}$ particle orbits and $1d_{3/2}$ $2s_{1/2}$, $1d_{5/2}$ hole orbits in ^{40}K nucleus. It is one-neutron (particle) outside the inert core ^{40}Ca and one proton (hole). The spectrum of this nucleus was calculated by using Eqs. (8) and (9) and from these matrix element plus (*s. p.* energy for particle – *s. p.* energy for hole) to obtain the energy we have the *s. p.* energy which are [29] the *s. p.* energy value [36, 37] where: $e_{P(particle)} = e1f_{7/2}$ = 8.329 MeV, $e_{H(hole)} = e1d_{5/2}$ = = –8.363 MeV, $e2s_{1/2}$ = –5.843 MeV and $e1d_{3/2}$ = = –3.973 MeV. The parameters A_0 = 0.8, A_1 = 0.9, B = 0.6 and C = –0.2. Table 2 and Fig. 2, *a* show comparison between theoretical and experiment values of ^{40}K nucleus.

Plotting energy levels of these states as a function of the corresponding angle $\theta_{1,2}$ determined according to Eq. (12) one can draw the smooth curve shown in Figs. 1 and 2 case (*b*) show the behaviour for even values $j_1 + j_2 + J = even$ and case (*c*) that for odd values of $j_1 + j_2 + J = odd$ effective interaction deduced from the data. The energy levels have been normalized to the centroid of each multiplet and are plotted against the angle $\theta_{1,2}$ defined in the text. Note that maximum *J* values correspond to minimum angle and vice versa. Curvature is a measure of a short-range attractive force. The lines are drawn to connect these points. The energy level seem to follow two universal functions which depend on the semi-classical coupling angles $\theta_{1,2}$ but are otherwise independent on *j*. For $j_1 \neq j_2$ several "typical" functions $\theta_{1,2}$ can be constructed which fit subsets of the data and differ in a predictable way.



a, *b*, *c*

Fig. 2.

Table 2. **Comparison between theoretical and experiment values**

J^π	Energy, MeV	Energy, MeV	J^π	Energy, MeV	Energy, MeV	J^π	Energy, MeV	Energy, MeV	J^π	Energy, MeV	Energy, MeV
^{40}Sc J^-	Theor.	Exp. [39]	^{40}K J^-	Theor.	Exp. [38]	^{40}Sc J^-	Theor.	Exp. [39]	^{40}K J^-	Theor.	Exp. [38]
4_1	0	0	4_1	0	0	3_2	2.992	–	1_1	2.091	2.103
3_1	0.03	0.034	3_1	0.03	0.029	3_3	3.05	3.03	3_3	2.189	2.29
2_1	0.77	0.772	2_1	0.801	0.800	4_3	3.07	–	4_2	2.296	2.397
5_1	0.89	0.893	5_1	0.881	0.891	5_2	3.35	3.49	5_2	2.35	2.423
4_2	2.27	2.37	2_2	1.999	2.047	6_1	3.42	3.49	6_1	2.447	2.558
1_1	2.911	2.933	3_2	2.011	2.069	2_2	3.5	3.9	4_3	2.569	2.575

5. Conclusion

In this work, the agreement between theoretical and experimental levels is satisfactory for excitation energies. There are many unconfirmed experimental energy levels confirmed by our calculations and new values for energy levels which were not specified in the experimental data. The choice of model space *MSDI* effective interactions are suitable in this mass region. Note that maximum *J* values correspond to minimum angle and vice versa. The general features of the bound-nucleon interaction appear consistent with those of theoretical matrix elements based on a number of short-range model forces where a delta-interaction force, due to its short-range attractive characteristics. This indicates that the shell model is very good to illustrate the nuclear structure for ^{40}Sc and ^{40}K nuclei.

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ЕНЕРГЕТИЧНІ РІВНІ ЯДЕР ^{40}Sc ТА ^{40}K ЗАЛЕЖНОСТІ ВІД НАПІВКЛАСИЧНОГО КУТА ЗВ'ЯЗКУ $\theta_{1,2}$ В МОДИФІКОВАНІЙ ПОВЕРХНЕВІЙ ДЕЛЬТА-ВЗАЄМОДІЇ

Оболонкову модель ядра з використанням модифікованої поверхневої дельта-взаємодії було застосовано в цій роботі для обчислення енергетичних рівнів ізобарних ядер ^{40}Sc та ^{40}K в стані частинка-дірка. Частинки знаходяться в стані ($1f_{7/2}$), тоді як дірки знаходяться в стані ($1d_{3/2}$, $1s_{1/2}$, $1d_{5/2}$). Загальний кутовий момент і парність визначено для можливих частинок і дірок у вищезазначених ядрах. Автори знайшли таким чином залежність між рівнями енергії та напівкласичним кутом зв'язку $\theta_{1,2}$ для різних орбіталей у межах конфігурації частинка-дірка. Можна відзначити, що рівні енергії наближено описуються двома універсальними функціями, які залежать від напівкласичного кута зв'язку $\theta_{1,2}$. Показано, що теоретичні розрахунки узгоджуються з експериментальними даними.

Ключові слова: оболонкова модель, енергетичні рівні, модифікована поверхнева дельта-взаємодія, ^{40}Sc, ^{40}K, частинка-дірка.

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ЭНЕРГЕТИЧЕСКИЕ УРОВНИ ЯДЕР ^{40}Sc И ^{40}K ЗАВИСИМОСТИ ОТ ПОЛУКЛАССИЧЕСКОГО УГЛА СВЯЗИ $\theta_{1,2}$ В МОДИФИЦИРОВАННОМ ПОВЕРХНОСТНОМ ДЕЛЬТА-ВЗАИМОДЕЙСТВИИ

Оболочечная модель ядра с использованием модифицированного поверхностного дельта-взаимодействия была применена в этой работе для вычисления энергетических уровней изобарных ядер ^{40}Sc и ^{40}K в состоянии частица-дырка. Частицы находятся в состоянии ($1f_{7/2}$), тогда как дырки находятся в состоянии ($1d_{3/2}$, $1s_{1/2}$, $1d_{5/2}$). Общий угловой момент и четность определены для возможных частиц и дырок у вышеупомянутых ядрах. Авторы нашли таким образом зависимость между уровнями энергии и полуклассическим углом связи $\theta_{1,2}$ для различных орбиталей в пределах конфигурации частица-дырка. Можно отметить, что уровни энергии приближенно описываются двумя универсальными функциями, которые зависят от полуклассического угла связи $\theta_{1,2}$. Показано, что теоретические расчеты согласуются с экспериментальными данными.

Ключевые слова: оболочечная модель, энергетические уровни, модифицированное поверхностное дельта-взаимодействие, ^{40}Sc, ^{40}K, частица-дырка.

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