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CALCULATION OF QUADRUPOLE DEFORMATION PARAMETER β_2 FROM REDUCED TRANSITION PROBABILITY $B(E2)\uparrow$ FOR $0_1^+ \to 2_1^+$ TRANSITION AT EVEN-EVEN $^{62-68}$Zn ISOTOPES

In this work the excited energy levels, reduced transition probabilities $B(E2)\uparrow$, intrinsic quadrupole moments, and deformation parameters have been calculated for $^{62-68}$Zn isotopes with neutrons number N = 32, 34, 36 and 38. NuSheIIX code has been applied for all energy states of fp-shell nuclei. Shell-model calculations for the zinc isotopes have been carried out with active particles distributed in the $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$ orbits outside doubly magic closed ^{56}Ni core nucleus. By using f5p model space and f5pvh interaction, the theoretical results have been obtained and compared with the available experimental results. The excited energies values, electric transition probability $B(E2)$, intrinsic quadrupole moment Q_0, and deformation parameters β_2 have appeared in complete agreement with the experimental values. As well as, the energy levels have been confirmed and determined for the angular momentum and parity of experimental values that have not been well established and determined experimentally. On the other hand, it has been predicted some of the new energy levels and electric transition probabilities for the $^{62-68}$Zn isotopes under this study which were previously unknown in experimental information.

Keywords: $B(E2)\uparrow$, ground-states, NuSheIIX code, deformation parameters.

1. Introduction

The structure of the atomic nuclei had witnessed a new contribution in 1948. Maria Goeppert Mayer and J. Hans D. Jensen introduced concept of spin-orbit coupling to describe effect of magic nucleus [1]. By the nuclear shell model (SM) there had been attempts to explain the behavior of nuclei in an identical manner such as the Bohr model for atoms, the main concept was to arrange the nucleons which are protons and neutrons, into a shell structure with major shells that include minor shells called subshells or orbitals [2]. Microscopic SM of nuclei assumes that nucleons have independent movement in a Hartree - Fock self-consistent potential which basically follows the mass distribution radial dependence and is necessary to be non-local due to the antisymmetrization required by the Pauli principle and contains a spin-orbit interaction to produce the correct magic numbers [3]. The nuclei with magic numbers of protons and/or neutrons are not only highly stable but also exhibit some additional characteristics in nuclei which can clearly show that the nuclear shell structure is associated with the independent-particle model for nuclei, each closed-shell configuration in this model gives a convenient first approximation which assumes the system under consideration includes a closed-shell core plus valence particles in a valence shell.

This approach very successfully explains the ground state properties of nuclei [4]. The SM has been greatly included in elucidating all properties of the nuclear levels: excitation energies, electromagnetic transition probabilities, and intrinsic quadrupole moments in the medium mass nuclei, particularly study nuclei in the vicinity of doubly magic (^{56}Ni) nucleus, the study of electromagnetic transition strengths in nuclei can supply available information on the ability of nuclear models to describe many nuclear properties accurately and systematically; this can help in understanding the underlying nuclear structure as well as study the properties of some (even-even) nuclei [5, 6]. Nuclear moments have been studied since the beginning of nuclear structure physics, the evaluation of nuclear quadrupole moments is more difficult and challenging than magnetic moment measurements, obviously, to understand the nuclear structure; measurement of the nuclei properties should be made over a large range of isospin or conduct a detailed investigation of some nuclei [7]. Then, the properties of a nucleus with several nucleons outside a closed shell will be described in a first approximation by an inert core (e.g., a doubly magic nucleus such as $^{56}_{28}$Ni in this present work) with a few nucleons which can move in a certain mixing configuration space and which interact with other nucleons via a residual interaction (particle-particle interactions). The essential inputs in calculations are the model space and effective interaction, which can probe the cogency of the model and parameterizations of the residual interaction via comparison to several experimental parameters (excitation energy, spin/parity, magnetic and quadrupole moment), the nuclear moments can be a good check if the parameterization and model space is appropriate, deviations from the model expectations might indicate the presence of configuration that can be mixed into other orbits (without taking into account the chosen model space) or better, parameterized residual interactions [8 - 10].

2. Theory

The necessity of nuclear shell-model calculations inputs are the model-space and effective interactions that represent the most adequate tool for reporting the diagonal matrix of the low energy of the Hamiltonian system, the Hamiltonian matrix sizes can be increased significantly with the increase of valence shells number of valence nucleons. Hence, the emphatic truncation of the configuration space is required [11]. Shell-model Hamiltonian in a microscopic approach may be to the shell structure of single-particle levels in a spherical potential, the construction start is from a realistic (N-N) potential by means of many-body perturbation techniques. This approach has long been a central topic of nuclear theory. The solution of the Schrödinger equation for any nuclear system can be given in the formula [12]

$$H|\psi_n\rangle = E_n|\psi_n\rangle, \tag{1}$$

where

$$H = H_0 + H_1, \tag{2}$$

$$H_0 = \sum_{i=1}^{A}\left(T_i + U_i\right), \tag{3}$$

$$H_1 = \sum_{i=1}^{A}\left(\sum_{j,i<j} V_{ij}^{NN} - U_i\right). \tag{4}$$

In order to separate the nuclear Hamiltonian, a one-body potential U_i has been introduced as the sum of a one-body term H_0, which describes the independent motion of the nucleons and the interaction H_1. Schrödinger equation solutions with H_0 are the single nucleon energies (SPE) in a central potential, as observed in single-particle (particles or holes) states outside a Double-closed shell (DCS) nucleus in its neighbors (DCS ± 1). The two-body matrix element (TBME) of the residual interaction H_1 constitutes the mutual interaction of the valence nucleons on the surface as observed in the DCS ± 2 neighbors' of a magic nucleus, nuclei with only a few particles outside closed shells have also spherical shapes in their ground states, the lowest states in the even-even nuclei are related to the quadrupole vibrations of the nuclear surface; they represent the degrees of freedom, which are easiest to excite, the spherical nuclear shape becomes less and less stable when the number of particles or holes in the unfilled shells increase [13].

The determination of the deviation from the spherical balance of the nuclear charge distribution is the electric quadrupole moment (Q_0). Therefore, electric quadrupole moment (Q_0) reflects one of the significant quantities to limit the shape of nuclei, the configurations of valence nucleons in unfilled orbits of the nucleus is one of the main causes of its deformation, another meaning the deformation occurs only when the shells of the neutron (N) and proton (Z) are somewhat overcrowded [14].

The intrinsic quadrupole moment (Q_0) is defined in the intrinsic frame of reference. It takes three values, zero for nuclei that have a spherically symmetric charge distribution, a negative value for oblate nuclei, and finally the positive value for prolate nuclei. It is given according to the formula [15 - 17]

$$Q_0 = \left(\frac{16\pi B(E2)\uparrow}{5e^2}\right)^{1/2}. \tag{5}$$

The symbols in the equation above can be defined as follow: $\left(B(E2)\uparrow\right)$ is electromagnetic quadrupole transition probability in Weisskopf units (W.u.) and e denotes the charge of the electron.

There is a relationship between $B(E2)$ in the unit of e^2b^2 and $B(E2)$ in Weisskopf unit (W.u.), this can be according to the following mathematical relationship:

$$B(E2)\uparrow e^2b^2 = 5.94\cdot 10^{-6}\cdot A^{4/3}\,B(E2)\,\text{W.u.}, \tag{6}$$

where b (barn) is the area unit and A is the mass number of the nucleus.

The reduced transition probability $B(E2)$ can be defined by

$$B(E2; i\to f) = \frac{< J_f\,|\,Q(\lambda)\,|\,J_i >^2}{2J_i+1}, \tag{7}$$

where $< J_f\,|\,Q(\lambda)\,|\,J_i >$ is the reduce matrix element. $B(E2;\ i\to f)$ depends upon the direction of the transition, for electromagnetic transitions J_i is the higher-energy initial state, the quadrupole transition probability $B(E2)\uparrow$ is related to the quadrupole deformation parameter β of nucleus shape in equilibrium as

$$B\left(E2;0_1^+\rightarrow 2_1^+\right)\uparrow=\left(\frac{3}{4}\pi e^2 Z R_0^2\right)^2\beta_2^2, \qquad (8)$$

where Z is the atomic number and R_0 is the average radius of the nucleus that is given by

$$R_0^2=0.0144\,A^{\frac{2}{3}}\,b. \qquad (9)$$

The main object of the current work is a shell model description of the exciting energies, $B\left(E2\,;0_1^+\rightarrow 2_1^+\right)\uparrow$ transitions probabilities, intrinsic quadrupole moments, deformation parameters, and in a selected set of even-even zinc isotopes.

3. Results and discussion

In this paper, the results are based on NuSheIIX-code for Windows [18]. This code uses a set of computer codes written by researcher Rae that are used to obtain exact energies, eigenvectors, and spectroscopic overlaps for low-lying states in shell model Hamiltonian matrix calculations with a very large basis dimension. It uses a J-coupled proton-neutron basis and J-scheme matrix dimensions of up to 100 million orders. The formulation comprises f5p-model space consisting of ($lp_{3/2}$, $0f_{5/2}$ and $lp_{1/2}$) shells above the ^{56}Ni nucleus With the interaction of f5pvh plus two Hamiltonian bodies derived by researchers Koops and Glaudemans of Ni and Cu isotopes that include calculations within the fp shell of orbitals ($lp_{3/2}$, $0f_{5/2}$ and $lp_{1/2}$) code NuSheIIX@MSU, single-particle energies are E ($1p_{3/2}$) = –9.4200 MeV, E ($0f_{5/2}$) = = –10.2700 MeV and E ($1p_{/2}$) = –9.0500 MeV. The two-body portion of the Hamiltonian consisted of experimental modifications of two-body matrix elements for delta surface (MSD1) of 63 matrix elements that were originally modified to accommodate the energy levels of the isotopes of nickel and copper [19, 20]. The energy levels selected for the parameter determination satisfy the conditions:

(i) The spin and parity (J^π) are sensibly convinced.

(ii) For each group (A, B, C, J, T) no more than the two states lying less at (A, B, C strength of interaction, J total angular momentum, and T isospin) [20] are included.

In this current work on Zn nuclei which is a direct extension of the work of authors Glaudemans and Koops on Ni and Cu nuclei and can be considered in one context as an independent and comprehensive test of Hamiltonian, we preferred the simpler and more direct approach. Extrapolating the results for Ni and Cu to Zn nuclei, we used the parameters for the strength of the interaction as in Table 1.

Table 1. **The values of the parameters resulting from various fits [20]**

Parameters	Values
A_1	(1) 0.53
A_0	(4) 0.34
B_1	(1) 0.42
B_0	–0.95 (8)
C_1	0
C_0	0

Parameters A_1, A_0, B_1, B_0, C_1, C_0 stand for the strength of the interaction when T =1 or 0 [20].

3.1. Energy levels

3.1.1. ^{62}Zn nucleus

Expected shell-model configurations for neutrons and protons in this nucleus primarily involve the ($lp_{3/2}$, $0f_{5/2}$, and $lp_{1/2}$) orbits above the (^{56}Ni). From Table 2, it can be expected a (0^+) ground state for the ^{62}Zn nucleus. The experimental data [21] and our results are shown in Table 2.

Table 2. **Theoretical and experimental exciting levels of ^{62}Zn nucleus by using f5pvh interaction [21]**

Calculations values		Experimental values		Calculations values		Experimental values	
J	Ex, MeV	J	Ex, MeV	J	Ex, MeV	J	Ex, MeV
0_1^+	0	0^+	0	5_5^+	4.760	–	–
2_1^+	0.870	2^+	0.953	0_8^+	4.770	–	–
2_2^+	1.830	2^+	1.804	6_4^+	4.827	–	–
4_1^+	2.048	4^+	2.186	1_{10}^+	4.831	–	–
0_2^+	2.203	0^+	2.341	5_6^+	4.942	(1^+)	4.895
2_3^+	2.247	–	–	7_1^+	4.950	(7^-)	4.904
3_1^+	2.312	3^+	2.384	0_9^+	5.050	(2^+)	5.050
4_2^+	2.553	4^+	2.743	6_5^+	5.235	(6^-)	5.131

Continuation of Table 2

Calculations values		Experimental values		Calculations values		Experimental values	
J	Ex, MeV	J	Ex, MeV	J	Ex, MeV	J	Ex, MeV
1_1^+	2.668	–	–	5_7^+	5.268	(0^+)	5.240
2_4^+	2.725	2^+	2.884	0_{10}^+	5.285	0^+	5.340
2_5^+	2.900	2^+	3.060	8_1^+	5.414	(8^+)	5.481
4_3^+	2.907	–	–	5_8^+	5.435	–	–
2_6^+	3.035	–	–	6_6^+	5.512	–	–
1_2^+	3.039	(0^+)	3.042	6_7^+	5.604	–	5.560
3_2^+	3.057	–	–	5_9^+	5.632	–	5.700
1_3^+	3.076	–	–	5_{10}^+	5.674	–	–
3_3^+	3.099	–	–	6_8^+	5.821	–	–
0_3^+	3.163	(2^+)	3.160	7_2^+	5.835	–	–
2_7^+	3.231	(1^+)	3.181	6_9^+	5.952	(1^+)	5.920
0_4^+	3.334	(4^+)	3.310	7_3^+	6.068	(9^-)	6.081
4_4^+	3.365	(1^-)	3.374	8_2^+	6.115	(8^-)	6.113
5_1^+	3.398	–	–	6_{10}^+	6.118	–	–
1_4^+	3.433	–	–	7_4^+	6.321	(8^+)	6.300
2_8^+	3.444	2^+	3.470	8_3^+	6.461	–	6.400
3_4^+	3.457	–	–	7_5^+	6.511	–	6.629
1_5^+	3.502	–	–	7_6^+	6.593	–	–
6_1^+	3.504	6^+	3.707	8_4^+	6.797	–	–
4_5^+	3.540	–	–	7_7^+	6.843	–	–
2_9^+	3.592	(2^+)	3.590	8_5^+	6.952	–	–
3_5^+	3.636	–	–	7_8^+	7.049	–	–
0_5^+	3.651	–	–	7_9^+	7.099	–	–
2_{10}^+	3.716	2^+	3.640	9_1^+	7.239	–	7.200
3_6^+	3.845	$(3^-, 4^+)$	3.730	7_{10}^+	7.320	–	–
4_6^+	3.880	–	–	10_1^+	7.366	(10^+)	7.500
1_6^+	3.895	–	–	8_6^+	7.575	(8^+)	7.540
0_6^+	3.933	–	–	8_7^+	7.882	–	–
3_7^+	3.950	$(3^-, 4^+)$	3.920	9_2^+	8.025	(9^+)	7.976
6_2^+	3.973	–	–	8_8^+	8.033	–	–
5_2^+	4.005	(1^+)	4.021	8_9^+	8.088	–	–
3_8^+	4.104	(4^+)	4.090	8_{10}^+	8.173	–	–
1_7^+	4.204	–	–	9_3^+	8.379	(6^+)	8.300
3_9^+	4.222	(3^-)	4.217	9_4^+	8.481	(10^+)	8.437
4_7^+	4.264	–	–	9_5^+	8.622	–	–
5_3^+	4.323	–	–	10_2^+	8.874	(11^+)	9.048
4_8^+	4.353	(4^+)	4.380	9_6^+	9.347	–	–
1_8^+	4.374	–	–	9_7^+	9.558	(12^+)	9.465
3_{10}^+	4.399	–	4.535	10_3^+	9.758	–	–
4_9^+	4.402	–	–	9_8^+	9.790	–	9.800
4_{10}^+	4.472	–	–	10_4^+	9.920	(12^+)	9.823
5_4^+	4.480	–	–	9_9^+	9.992	(13^+)	9.960
1_9^+	4.484	(1^+)	4.448	9_{10}^+	10.113	–	10.300
6_3^+	4.599	(7^-)	4.600	10_5^+	10.967	–	10.800
0_7^+	4.693	(0^+)	4.620	10_6^+	11.939	(16^+)	11.961

The experimental energies {0.953, 1.804, 2.186, 2.341, 2.384, 2.743, 2.884, 3.064, 3.470, 3.707, 3.640 and 5.340} MeV have a good congruence with the expected theoretical values at states $\{2_1^+, 2_2^+, 4_1^+, 0_2^+, 3_1^+, 4_2^+, 2_4^+, 2_5^+, 2_8^+, 6_1^+, 2_{10}^+$ and $0_{10}^+\}$, respectively. Theoretically: the energy states $\{1_2^+, 0_3^+, 2_7^+, 0_4^+, 4_4^+, 2_9^+, 3_6^+, 3_7^+, 5_2^+, 3_8^+, 3_9^+, 4_8^+, 1_9^+, 6_3^+, 0_7^+, 5_6^+, 7_1^+, 0_9^+, 6_5^+, 5_7^+, 8_1^+, 6_9^+, 7_3^+, 8_2^+, 7_4^+, 10_1^+, 8_6^+, 9_2^+, 9_3^+, 9_4^+, 10_2^+, 9_7^+, 10_4^+, 9_6^+$ and $10_6^+\}$ have been confirmed for experimental energy values {3.042, 3.160, 3.181, 3.310, 3.374, 3.590, 3.730, 3.920, 4.021, 4.090, 4.217, 4.380, 4.448, 4.600, 4.620, 4.895, 4.904, 5.050, 5.131, 5.240, 5.481, 5.920, 6.081, 6.113, 6.300, 7.500, 7.540, 7.976, 8.300, 8.437, 9.048, 9.465, 9.823, 9.960 and 11.961} MeV for the uncertain practically levels at spins and parities. Experimentally: the energies {4.535, 5.560, 5.700, 6.400, 6.629, 7.200, 9.800, 10.300 and

10.800} MeV have been predicted in theoretical results with the states {3_{10}^+, 6_7^+, 5_9^+, 8_3^+, 7_6^+, 9_1^+, 9_8^+, 9_{10}^+ and 10_5^+} these energies had not been predicted previously at spins and parities. In studied theoretical results, new energy levels have been expected for states; spins and parties such as {2.247; (2_3^+), 2.668; (1_1^+), 2.907; (4_3^+), 3.035; (2_6^+), (3.039 to 3.099); for states (3_2^+ to 3_4^+), 3.398; (5_1^+), 3.433; (1_4^+), 3.457; (3_4^+), 3.502; (1_5^+), 3.540; (4_5^+), 3.636; (3_5^+), 3.651; (0_5^+), (3.880 to 3.933) for states (4_6^+ to 0_6^+), 3.973; (6_2^+), 4.204; (1_7^+), 4.264; (4_7^+), 4.323; (5_3^+), 4.374; (1_8^+), (4.402 to 4.480); for states (4_9^+ to 5_4^+), (4.760 to 4.831); for states (5_5^+ to 1_{10}^+), 5.435; (5_8^+), 5.512; (6_6^+), (5.674 to 5.835) for states; (5_{10}^+ to7_2^+), 6.118; (6_{10}^+), 6.511; (7_5^+), (6.797 to 7.099) for states; (8_4^+ to 7_9^+), 7.320; (7_{10}^+), 7.882;(8_7^+), (8.033 to 8.173) for states; (8_8^+ to 8_{10}^+), 8.622; (9_5^+), 9.347; (9_6^+) and 9.758; (10_3^+)}, respectively, these energies and states had not been predicted previously at available experimental information.

3.1.2. ^{64}Zn nucleus

This nucleus contains eight nucleons (two protons and six neutrons) outside the ^{56}Ni core nucleus for configurations spaces (lp$_{3/2}$, 0f$_{5/2}$ and lp$_{1/2}$). In Table 3, the experimental energies [22] are given with states such as {1.799; (2^+), 2.793; (2^+), 3.077; (4^+) and 4.236; (6^+)} MeV, these have a good correspondance with the predicted theoretical states and energies {1.832; (2_2^+), 2.774; (2_4^+), 2.927; (2_5^+), 2.966; (4_4^+) and 4.260; (6_4^+)}. Theoretically, it has been expected that the energies {0.938; (2_1^+), 1.719; (0_2^+), 2.201; (4_1^+), 2.378; (0_3^+), 3.061; (1_3^+), 3.180; (1_4^+), 3.524; (4_7^+) and 3.843; (5_2^+)} are rather in a good agreement with the experimental data {0.991; (2^+), 1.910; (0^+), 2.306; (4^+), 2.609; (0^+), 3.186; (1^+), 3.261; (1^+), 3.552; (4^+) and 3.852; (5^+)}, respectively. In the studied calculations, it has been predicted confirmation for the states spins and parities as {2_6^+, 3_3^+, 3_4^+, 0_4^+, 5_1^+, 2_8^+, 0_5^+, 4_6^+, 2_9^+, 1_5^+, 2_{10}^+, 0_6^+, 3_6^+, 3_7^+, 4_8^+, 3_8^+, 4_8^+, 3_9^+, 4_9^+, 0_7^+, 1_8^+, 5_3^+, 3_{10}^+, 4_{10}^+, 1_9^+, 1_{10}^+, 5_5^+, 5_6^+, 5_7^+, 6_6^+, 5_9^+, 6_7^+, 5_{10}^+, 6_9^+, 7_6^+, 8_3^+, 8_4^+, 8_7^+, 8_{10}^+, 10_1^+, 10_2^+, 9_6^+, 9_7^+, 10_3^+, 11_1^+, 10_6^+, 10_7^+, 10_{10}^+, 11_2^+,} for experimental energies {3.094, 3.196, 3.205, 3.240, 3.285, 3.297, 3.321, 3.415, 3.452, 3.458, 3.500, 3.606, 3.620, 3.627, 3.628, 3.718, 3.850, 3.863, 3.880, 3.880, 3.889, 3.932, 3.952, 4.039, 4.076, 4.140, 4.305, 4.420, 4.467, 4.668, 4.786, 4.823, 5.902, 5.121, 5.936, 6.031, 6.377, 6.998, 7.212, 7.556, 8.181, 8.303, 8.426, 8.580, 9.666, 9.804, 9.948, 10.460 and 11.626} MeV, respectively. In this nucleus, new energies have been expected (in MeV units) with spins and parities as {2.080; (2_3^+), (2.488 to 2.768); (4_2^+ to 3_2^+), 2.843; (4_3^+), 3.041; (2_7^+), 3.044; (1_2^+), 3.138; (4_5^+), 3.540; (1_6^+), 3.868; (6_2^+), 4.015; (0_8^+), 4.113; (6_3^+), 6.141; (7_7^+), 6.171; (7_8^+), 6.397; (8_5^+); 6.471; (7_{10}^+), (6.909 to 7.098); (9_1^+ to 8_9^+), 7.685; (9_3^+), 8.087; (9_5^+), (8.620 to 9.484); (9_8^+ to 10_5^+), 10.246; (10_8^+), 10.329; (10_9^+). These energies with states are not experimentally known yet. Experimental energies (in MeV units) as {3.586, 3.698, 4.154, 4.181, (4.504 to 4.638), 4.761, 5.110, (5.337 to 5.792), 6.300, 6.830, 7.380 and 7.900 in the studied calculations specified with states spins and parities {1_6^+, 6_1^+, 0_9^+, 5_4^+, (0_{10}^+ to 5_8^+), 7_1^+, 6_8^+, (6_{10}^+ to 8_2^+), 7_9^+, 8_6^+, 9_2^+ and 9_4^+} respectively. The experimental value 3.538 MeV has been specified in these studied results with state 3_5^+ while in experimental data specified with state {2 to 6}.

Table 3. **Theoretical and experimental exciting levels of ^{64}Zn nucleus by using f5pvh interaction [22]**

Calculations values		Experimental values		Calculations values		Experimental values	
J	Ex, MeV	J	Ex, MeV	J	Ex, MeV	J	Ex, MeV
0_1^+	0	0^+	0	5_6^+	4.335	$(3, 4, 5)^+$	4.420
2_1^+	0.938	2^+	0.991	5_7^+	4.439	(0^+)	4.467
0_2^+	1.719	0^+	1.910	0_{10}^+	4.497	–	4.504
2_2^+	1.832	2^+	1.799	6_5^+	4.551	–	4.556
2_3^+	2.080	–	–	5_8^+	4.659	–	4.638
4_1^+	2.201	4^+	2.306	6_6^+	4.667	(6^-)	4.668
0_3^+	2.378	0^+	2.609	7_1^+	4.765	–	4.761
3_1^+	2.433	3^+	2.979	5_9^+	4.813	(3^+, 4^+, 5^+)	4.786
4_2^+	2.488	–	–	6_7^+	4.829	(5, 6, 7)	4.823
1_1^+	2.712	–	–	5_{10}^+	4.884	(3^+, 4^+, 5^+)	4.902
3_2^+	2.768	–	–	6_8^+	5.119	–	5.110
2_4^+	2.774	2^+	2.793	6_9^+	5.123	(1^+, 2^+, 3^+)	5.121
4_3^+	2.843	–	–	6_{10}^+	5.335	–	5.337
2_5^+	2.927	2^+	3.005	7_2^+	5.413	–	5.413
4_4^+	2.966	4^+	3.077	8_1^+	5.526	–	5.530
2_6^+	2.972	$(3)^+$	3.094	7_3^+	5.565	–	5.545
2_7^+	3.041	–	–	7_4^+	5.612	–	5.613

Continuation of Table 3

Calculations values		Experimental values		Calculations values		Experimental values	
J	Ex, MeV	J	Ex, MeV	J	Ex, MeV	J	Ex, MeV
1_2^+	3.044	–	–	7_5^+	5.749	–	5.760
1_3^+	3.061	1^+	3.186	8_2^+	5.796	–	5.792
3_3^+	3.081	(2, 3)	3.196	7_6^+	5.992	(8^+)	5.936
3_4^+	3.118	$(3)^+$	3.205	8_3^+	6.005	(8^+)	6.031
4_5^+	3.138	–	–	7_7^+	6.141	–	–
0_4^+	3.141	$(0)^+$	3.240	7_8^+	6.171	–	–
1_4^+	3.180	1	3.261	7_9^+	6.289	–	6.300
5_1^+	3.210	$(1^-:5^-)$	3.285	8_4^+	6.377	(9^-)	6.377
2_8^+	3.253	$(2)^+$	3.297	8_5^+	6.397	–	–
0_5^+	3.372	(1)	3.321	7_{10}^+	6.471	–	–
4_6^+	3.388	$(1^-:5^-)$	3.415	8_6^+	6.789	–	6.830
2_9^+	3.391	$(1, 2^+)$	3.452	8_7^+	6.857	(11^-)	6.998
1_5^+	3.422	(2, 3)	3.458	9_1^+	6.909	–	–
2_{10}^+	3.448	(2^+)	3.500	8_8^+	6.925	–	–
3_5^+	3.514	(2 to 6)	3.538	8_9^+	7.098	–	–
4_7^+	3.524	4^+	3.552	8_{10}^+	7.202	(11^-)	7.212
1_6^+	3.540	–	3.586	9_2^+	7.424	–	7.380
0_6^+	3.600	(≤ 4)	3.606	9_3^+	7.685	–	–
3_6^+	3.613	(2 to 6)	3.620	10_1^+	7.844	(10^-)	7.556
1_7^+	3.618	$(0^+, 6^-)$	3.627	9_4^+	7.885	–	7.900
3_7^+	3.639	(4^+)	3.628	10_2^+	8.034	(10^-)	8.181
6_1^+	3.722	–	3.698	9_5^+	8.087	–	–
4_8^+	3.742	$(0^+:4^+)$	3.718	9_6^+	8.350	(12^-)	8.303
3_8^+	3.842	$(\le 3)(\quad ^+)$	3.850	9_7^+	8.427	(11^-)	8.426
5_2^+	3.843	5^+	3.853	10_3^+	8.531	(12^+)	8.580
3_9^+	3.859	$(2^+:6^+)$	3.863	9_8^+	8.620	–	–
6_2^+	3.868	–	–	9_9^+	8.733	–	–
4_9^+	3.875	$(0^+:4^+)$	3.880	9_{10}^+	8.831	–	–
0_7^+	3.893	$(0^+:4^+)$	3.880	10_4^+	9.250	–	–
1_8^+	3.904	$(2^+, 3, 4^+)$	3.889	10_5^+	9.484	–	–
0_8^+	4.015	–	–	11_1^+	9.569	(14)	9.666
5_3^+	4.033	(4, 5)	3.932	10_6^+	9.928	–	–
3_{10}^+	4.042	$(3^+, 4)$	3.952	10_7^+	10.073	(11^-)	9.804
4_{10}^+	4.052	$(0^+:4^+)$	4.039	10_8^+	10.246	(12^-)	9.948
1_9^+	4.062	$(5)^+$	4.076	10_9^+	10.329	–	–
1_{10}^+	4.088	$(1^+, 2^+)$	4.140	10_{10}^+	10.432	(13^-)	10.460
6_3^+	4.113	–	–	11_2^+	11.755	(15^-)	11.626
5_4^+	4.143	–	4.154	–	–	–	–
0_9^+	4.256	–	4.181	–	–	–	–
6_4^+	4.260	6^+	4.236	–	–	–	–
5_5^+	4.280	$(1^-, 5^-)$	4.305	–	–	–	–

3.1.3. ^{66}Zn nucleus

This nucleus is described as containing ten nucleons (two protons and eight neutrons) over the close core ^{56}Ni distribution at $(lp_{3/2}$, $0f_{5/2}$ and $lp_{1/2})$ orbits. In Table 4, theoretically: agreement has been predicted for energies in MeV units as {1.022; (2_1^+), 1.697; (2_2^+), 2.368; (4_1^+), 2.756; (4_3^+), 2.979; (0_3^+), 2.990; (1_2^+), 3.050; (4_4^+), 3.203; (1_4^+), 3.239; (2_8^+), 3.520; (0_5^+) and 4.088; (1_9^+)}, respectively with experimental energies [23] {1.039, 1.872, 2.451, 2.765, 3.105, 3.077, 3.228, 3.331, 3.531 and 4.085} at the same states spins and parties from studied calculations. The theoretical states $\{3_1^+, 0_4^+, 5_2^+, 4_8^+, 4_9^+, 6_1^+, 5_4^+, 4_{10}^+, 1_6^+, 6_3^+, 7_3^+, 7_7^+, 8_7^+$ and $9_2^+\}$ have been affirmed for experimental energies {2.703, 3.030, 3.709, 3.882, 3.969, 4.075, 4.119, 4.182, 4.223, 4.251, 5.464, 6.292, 6.850 and 7.550}. In the studied results the energy value 3.380 MeV has been determined with state 1_5^+ while in experimental value 3.432 MeV has been specified with state $\{1,2^-\}$, as well as theoretical value 3.760; (0_6^+) MeV has been determined for energy value 3.731 in experimental information. Theoretical states $\{2_7^+, 3_4^+, 1_7^+, 3_7^+, 3_8^+, 1_8^+, 6_2^+, 0_8^+, 5_5^+, 0_{10}^+, (5_7$ to $7_2^+), 8_2^+, 7_9^+, 8_9^+$ and $10_6^+\}$ for experimental energies in MeV units have been determined as {3.241, 3.523, 3.731, 3.806, 3.874, 3.924, 4.081, 4.108, 4.321, 4.454 (4.527 to 5.352) 6.000, 6.419, 7.170 and 11.514} these have not been known previously in the states (spins and parities). The energies and states {2.227; (2_3^+), 2.536; (4_2^+), 2.777; (1_1^+), 2.869; (2_5^+), 2.916; (3_2^+), 3.004; (2_6^+), 3.078; (1_3^+), 3.127; (3_3^+), 3.222; (4_5^+), 3.350; (5_1^+), 3.434; (2_9^+), 3.519; (4_6^+), 3.592; (3_5^+), 3.614; (1_6^+), 3.911; (3_9^+), 3.932; (0_7^+), 4.027; (5_3^+), 4.084;($3_{10}^+)$, 4.214; (0_9^+), 4.379; (6_4^+), 4.385; (5_6^+), 4.476; (6_5^+), (5.747 to 5.943) for states; $(7_4^+$ to $7_5^+)$, 6.094; (7_6^+), 6.164; (8_3^+), 6.343; (8_4^+), 6.405; (7_8^+), (6.593 to 7.003) for states; $(8_5^+$ to $8_6^+)$, 7.161; (8_8^+), 7.300; (9_1^+) and (7.703 to 10.470) for states; $(8_{10}^+$ to $10_5^+)$} have been predicted theoretically, the energies and states have not been previously specified in experimental information.

Table 4. **Theoretical and experimental exciting levels of ^{66}Zn nucleus by using f5pvh interaction [23]**

Calculations values		Experimental values		Calculations values		Experimental values	
J	Ex, MeV	J	Ex, MeV	J	Ex, MeV	J	Ex, MeV
0_1^+	0	0^+	0	0_9^+	4.214	–	–
2_1^+	1.022	2^+	1.039	1_{10}^+	4.235	(1^-)	4.223
2_2^+	1.697	2^+	1.872	6_3^+	4.238	(7^-)	4.251
2_3^+	2.227	–	–	5_5^+	4.352	–	4.321
4_1^+	2.368	4^+	2.451	6_4^+	4.379	–	–
3_1^+	2.404	(3)	2.703	5_6^+	4.385	–	–
4_2^+	2.536	–	–	0_{10}^+	4.404	–	4.454
4_3^+	2.756	4^+	2.765	6_5^+	4.476	–	–
2_4^+	2.772	2^+	2.780	5_7^+	4.563	–	4.527
1_1^+	2.777	–	–	5_8^+	4.669	–	4.680
0_2^+	2.842	–	–	6_6^+	4.752	–	4.758
2_5^+	2.869	2^+	2.938	5_9^+	4.803	–	4.832
3_2^+	2.916	–	–	6_7^+	4.872	–	4.875
0_3^+	2.979	(0^+)	3.030	5_{10}^+	4.921	–	4.918
1_2^+	2.990	–	–	7_1^+	5.028	–	5.025
2_6^+	3.004	–	–	6_8^+	5.133	–	5.124
4_4^+	3.050	4^+	3.077	6_9^+	5.282	–	5.285
1_3^+	3.078	–	–	6_{10}^+	5.342	–	5.331
3_3^+	3.127	–	–	7_2^+	5.345	–	5.352
0_4^+	3.152	0^+	3.105	7_3^+	5.463	(9^-)	5.464
1_4^+	3.203	1^+	3.228	7_4^+	5.747	–	–
4_5^+	3.222	–	–	8_1^+	5.800	–	–
2_7^+	3.231	–	3.241	7_5^+	5.943	–	–
2_8^+	3.239	2^+	3.331	8_2^+	6.086	–	6.000
5_1^+	3.350	–	–	7_6^+	6.094	–	–
1_5^+	3.380	$1, 2^-$	3.432	8_3^+	6.164	–	–
2_9^+	3.434	–	–	7_7^+	6.263	(10^+)	6.292
4_6^+	3.438	–	–	8_4^+	6.343	–	–
3_4^+	3.481	–	3.523	7_8^+	6.405	–	–
4_7^+	3.519	–	–	7_9^+	6.453	–	6.419
0_5^+	3.520	0^+	3.531	8_5^+	6.593	–	–
3_5^+	3.592	–	–	7_{10}^+	6.597	–	–
1_6^+	3.614	–	–	8_6^+	7.003	–	–
2_{10}^+	3.621	2^+	3.670	8_7^+	7.071	(8^+)	6.850
3_6^+	3.657	$1^-, 2^+, 3^+$	3.689	8_8^+	7.161	–	–
5_2^+	3.708	(5)	3.709	8_9^+	7.187	–	7.170
1_7^+	3.743	–	3.731	9_1^+	7.300	–	–
0_6^+	3.760	+	3.738	9_2^+	7.560	(6^+)	7.550
3_7^+	3.783	–	3.806	8_{10}^+	7.703	–	–
3_8^+	3.868	–	3.874	9_3^+	7.975	–	–
4_8^+	3.877	$(2)^+$	3.882	10_1^+	8.188	–	–
1_8^+	3.903	–	3.924	9_4^+	8.206	–	–
3_9^+	3.911	–	–	9_5^+	8.534	–	–
0_7^+	3.932	–	–	10_2^+	8.558	–	–
4_9^+	3.968	(4^+)	3.969	9_6^+	8.678	–	–

Continuation of Table 4

Calculations values		Experimental values		Calculations values		Experimental values	
J	Ex, MeV	J	Ex, MeV	J	Ex, MeV	J	Ex, MeV
6_1^+	3.968	(6^-)	4.075	9_7^+	8.927	–	–
5_3^+	4.027	–	–	9_8^+	9.206	–	–
6_2^+	4.083	–	4.081	9_9^+	9.312	–	–
3_{10}^+	4.084	–	–	9_{10}^+	9.358	–	–
1_9^+	4.088	1^+	4.085	10_3^+	9.481	–	–
0_8^+	4.113	–	4.108	10_4^+	10.289	–	–
5_4^+	4.153	(1^-)	4.119	10_5^+	10.470	–	–
4_{10}^+	4.164	(6^+)	4.182	10_6^+	11.543	–	11.514

3.1.4. ^{68}Zn nucleus

In this nucleus valence nucleons {two protons and ten neutrons} outside close core ^{56}Ni distribution at (lp$_{3/2}$, 0f$_{5/2}$ and lp$_{1/2}$) orbits. The predicted excited energy calculations values in MeV units are in Table 5 as follow. The levels {1.126; (2_1^+), 2.815; (2_4^+) and 3.936; (3_6^+)} have been very corresponding with experimental excited energy values [24] as {1.077; (2^+), 2.821; (2^+) and 3.935; (3^+)}. While the levels {1.603; (2_2^+), 2.489; (4_1^+) and 3.184; (0_3^+)} are in rather agreement with experimental excited energy values as {1.883; (2^+), 2.417; (4^+) and 3.102; (0^+)}. The states {4_3^+, 1_2^+, 0_4^+, 2_8^+, 2_9^+, 0_5^+, 4_6^+, 3_7^+, 3_9^+, 4_9^+, 4_{10}^+ and 1_8^+} have been affirmed for experimental excited energies{2.959, 3.186, 3.664, 3.709, 3.942, 3.989, 4.027 and 4.284}. We define the states {3_3^+, 1_7^+, 1_9^+ and 1_{10}^+} for experimental energies {3.496; (3^+, 4^+), 4.414; (1^+, 2^+), 4.732; (1^+, 2^+) and 4.910; (1^+, 2^+)} MeV, respectively. Assigned the energies with states as {2.675; (4_2^+), 2.742; (2_3^+), 2.890; (0_2^+), 2.980; (1_1^+), 3.079; (4_4^+), 3.089; (2_5^+), 3.308; (1_3^+), 3.592; (2_7^+), 3.746; (3_4^+), (3.825 to 3.917); (4_5^+ to 5_2^+), (4.099 to 4.204); (1_6^+ to 5_3^+), (4.524 to 4.594); (3_{10}^+ to 0_8^+), 4.921; (0_9^+), 5.079; (5_6^+), 5.530; (6_4^+), (5.804 to 8.678); (6_6^+ to 7_{10}^+)}, in the available experimental information, energies and states had not been previously known. These states {3_1^+, 3_2^+, 3_1^+, 2_6^+, 1_4^+, 1_5^+, 2_{10}^+, 0_6^+, 4_8^+, 0_7^+, 3_7^+, 4_{10}^+, 5_5^+ (0_{10}^+ to 7_1^+) and (5_9^+ to 6_5^+)} had been specified for the experimental energies: {2.510, 2.955, 3.454, 3.487, 3.929, 4.061, 4.096, 4.229, 4.252, 4.284, 4.680, 4.982 (5.146 to 5.565) and (5.610 to 5.990)} which have not been unspecified at the states spins and parities experimentally.

Table 5. **Theoretical and experimental excited levels of ^{68}Zn nucleus by using f5pvh interaction [24]**

Calculations values		Experimental values		Calculations values		Experimental values	
J	Ex, MeV	J	Ex, MeV	J	Ex, MeV	J	Ex, MeV
0_1^+	0	0^+	0	3_8^+	4.335	–	4.325
2_1^+	1.126	2^+	1.077	1_7^+	4.413	$1^+, 2^+$	4.414
2_2^+	1.603	2^+	1.883	3_9^+	4.460	$(1^+, 2^+)$	4.444
4_1^+	2.489	4^+	2.417	6_2^+	4.492	$(1, 2^+)$	4.496
3_1^+	2.508	–	2.510	4_9^+	4.501	(2^+)	4.512
4_2^+	2.675	–	–	3_{10}^+	4.524	–	–
2_3^+	2.742	–	–	6_3^+	4.532	–	–
2_4^+	2.815	2^+	2.821	5_4^+	4.533	–	–
0_2^+	2.890	–	–	0_8^+	4.594	–	–
3_2^+	2.938	–	2.955	1_8^+	4.613	(1^-)	4.608
1_1^+	2.980	–	–	4_{10}^+	4.713	–	4.680
4_3^+	3.038	(4^+)	2.959	1_9^+	4.735	$1, 2^+$	4.732
4_4^+	3.079	–	–	1_{10}^+	4.895	$1, 2^+$	4.910
2_5^+	3.089	–	–	0_9^+	4.921	–	–
0_3^+	3.184	0^+	3.102	5_5^+	5.041	–	4.982
1_2^+	3.212	$(1, 2^+)$	3.186	5_6^+	5.079	–	–
1_3^+	3.308	–	–	0_{10}^+	5.144	–	5.146
2_6^+	3.458	–	3.451	5_7^+	5.193	–	5.187
1_4^+	3.480	–	3.487	6_4^+	5.267	–	5.283
3_3^+	3.502	$3^+, 4^+$	3.496	5_8^+	5.304	–	5.420
2_7^+	3.592	–	–	7_1^+	5.454	–	5.565
0_4^+	3.671	$(1, 2)^+$	3.664	6_5^+	5.530	–	–
2_8^+	3.684	(2^+)	3.709	5_9^+	5.584	–	5.610
3_4^+	3.746	–	–	5_{10}^+	5.612	–	5.693

Continuation of Table 5

Calculations values		Experimental values		Calculations values		Experimental values	
J	Ex, MeV	J	Ex, MeV	J	Ex, MeV	J	Ex, MeV
4_5^+	3.825	–	–	6_6^+	5.724	–	–
5_1^+	3.845	–	–	6_7^+	5.804	–	–
3_5^+	3.917	–	–	6_8^+	5.957	–	–
5_2^+	3.917	–	–	7_2^+	5.980	–	–
1_5^+	3.921	–	3.929	6_9^+	6.187	–	–
3_6^+	3.936	3^+	3.935	8_1^+	6.270	–	–
2_9^+	3.937	(7^-)	3.942	6_{10}^+	6.411	–	–
0_5^+	3.972	$(1^-, 2^+)$	4.027	8_2^+	6.642	–	–
4_6^+	3.983	(2^+)	4.061	7_3^+	6.652	–	–
2_{10}^+	3.991	–	4.096	7_4^+	6.723	–	–
0_6^+	4.081	–	4.102	7_5^+	6.944	–	–
1_6^+	4.099	–	–	7_6^+	7.103	–	–
4_7^+	4.099	–	–	7_7^+	7.615	–	–
6_1^+	4.157	–	–	7_8^+	7.986	–	–
5_3^+	4.204	–	–	8_3^+	8.171	–	–
4_8^+	4.229	–	4.229	7_9^+	8.248	–	–
0_7^+	4.255	–	4.252	8_4^+	8.640	–	–
3_7^+	4.256	$(2, 3^+)$	4.284	7_{10}^+	8.678	–	–

3.2. Reduced electric quadrupole transition probabilities *B*(*E*2)↑

In this work the electric quadrupole transition probabilities have been calculated for $^{62-68}$Zn nuclei within the framework of the nuclear shell model. Transition levels and up electric quadrupole reduced transition probabilities *B*(*E*2)↑ for the ground state band from {(0^+ to 2^+),(2^+ to 4^+), (4^+ to 6^+), (6^+ to 8^+) and (8^+ to 10^+)} of even-even $^{62-68}$Zn isotopes are presented in Table 6. The electric quadrupole transition probabilities illustrate a sensitive test for the most modern effective interactions that have been gained to describe fp-shell nuclei. The transition probability has been calculated in this work by using the harmonic oscillator potential (HO, b), where b > 0 for each in-band transition. Core polarization effects have been included by choosing the effective charges for protons {e_p = 1.035e, 1.208e,1.140e and 1.102e} and for neutrons {e_n = 0.502e, 0.600e, 0.624e and 0.725e} for nuclei $^{62-68}$Zn nuclei, respectively. In Table 6, the electric quadrupole transition probabilities are calculated by using f5pvh effective interaction. All of the calculated results for electric quadrupole transition probabilities are reasonably consistent with available experimental data [21 - 24].

Table 6. **Reduced transition probability *B*(*E*2)↑ in even $^{62-68}$Zn nuclei [21 - 24]**

Nuclei	Valence nucleons	J^+	E_{exp}, MeV	Spin sequences	$B(E2)\uparrow_{exp}$, W.u.	$B(E2)\uparrow_{cal}$, W.u.
^{62}Zn	6	2	9.538	$0_1^+ \to 2_1^+$	84 ± (40)	83.972
		4	2.186	$2_1^+ \to 4_1^+$	46.8 ± (13 – 22)	39.509
		6	3.707	$4_1^+ \to 6_1^+$	27.4 ± (4)	24.917
		8	5.481	$6_1^+ \to 8_1^+$	10.330 ± (26 – 52)	21.027
		10	–	$8_1^+ \to 10_1^+$	–	14.290
^{64}Zn	8	2	0.991	$0_1^+ \to 2_1^+$	100 ± (30)	100.02
		4	2.306	$2_1^+ \to 4_1^+$	21.96 ± (9)	47.63
		6	3.698	$4_1^+ \to 6_1^+$	–	32.890
		8	5.530	$6_1^+ \to 8_1^+$	–	22.70
		10	–	$8_1^+ \to 10_1^+$	–	5.84
^{66}Zn	10	2	1.039	$0_1^+ \to 2_1^+$	87.5 ± (20)	87.481
		4	2.451	$2_1^+ \to 4_1^+$	32.94	38.46
		6	4.182	$4_1^+ \to 6_1^+$	21.66 ± (9)	27.97
		8	–	$6_1^+ \to 8_1^+$	–	12.45
		10	–	$8_1^+ \to 10_1^+$	–	12.07

Continuation of Table 6

Nuclei	Valence nucleons	J^+	E_{exp}, MeV	Spin sequences	$B(E2)\uparrow_{exp}$, W.u.	$B(E2)\uparrow_{cal}$, W.u.
^{68}Zn	12	2	1.077	$0_1^+ \to 2_1^+$	73.45 ± (95)	73.45
		4	2.417	$2_1^+ \to 4_1^+$	19.44 ± (22)	24.87
		6	–	$4_1^+ \to 6_1^+$	–	19.09
		8	–	$6_1^+ \to 8_1^+$	–	9.41
		10	–	$8_1^+ \to 10_1^+$	–	9.41

3.3. Intrinsic quadrupole moments and deformation parameters

The intrinsic quadrupole moments (Q_0) of the $^{62\text{-}68}$Zn nuclei have been calculated by Eq. (5) for even-even $^{62\text{-}68}$Zn isotopes and presented in Table 7, which have been compared with the experimental values. The calculated intrinsic quadrupole moments of shell-model calculations are in a good agreement with the previous experimental results [25], as well as the deformation parameters of nuclei with proton $Z = 30$ and even neutron ($N = 30 - 38$) are obtained by using Eq. (8) and presented in Table 7. The calculated deformation parameters associated with the framework of shell model have been compared with the corresponding previous experimental results [25, 26].

Table 7. **Intrinsic quadrupole moments and deformation parameters for even $^{62\text{-}68}$Zn nuclei [25, 26]**

Nuclei	Spin sequences	$B(E2)\uparrow_{exp}$ (e^2b^2)	$B(E2)\uparrow_{cal}$ (e^2b^2)	$\beta_{2(cal)}$	$\beta_{2(exp)}$ [25]	$\beta_{2(exp)}$ [26]	$Q_{0(cal)}$ (b)	$Q_{0(exp)}$ (b)
^{62}Zn	$0_1^+ \to 2_1^+$	0.1224 ± 58	0.122	0.216	0.216(52)	0.218(8)	1.107	1.116(41)
	$2_1^+ \to 4_1^+$	0.0682 ± 51	0.0576					
	$4_1^+ \to 6_1^+$	0.0399 ± 22	0.0363					
	$6_1^+ \to 8_1^+$	0.0150 ± 29	0.0306					
	$8_1^+ \to 10_1^+$	–	0.0208					
^{64}Zn	$0_1^+ \to 2_1^+$	0.1521 ± 45	0.1521	0.236	0.2342(32)	0.242(11)	1.236	1.27(6)
	$2_1^+ \to 4_1^+$	0.0349 ± 13	0.0724					
	$4_1^+ \to 6_1^+$	–	0.0500					
	$6_1^+ \to 8_1^+$	–	0.0345					
	$8_1^+ \to 10_1^+$	–	$0.88 \cdot 10^{-2}$					
^{66}Zn	$0_1^+ \to 2_1^+$	0.1386	0.1386	0.221	0.2198(26)	0.218(8)	1.180	1.164(43)
	$2_1^+ \to 4_1^+$	0.0522	0.0609					
	$4_1^+ \to 6_1^+$	0.0343	0.0443					
	$6_1^+ \to 8_1^+$	–	0.0197					
	$8_1^+ \to 10_1^+$	–	0.0191					
^{68}Zn	$0_1^+ \to 2_1^+$	0.1211	0.1211	0.202	0.2015(17)	0.205(12)	1.103	1.11(7)
	$2_1^+ \to 4_1^+$	0.0321	0.0410					
	$4_1^+ \to 6_1^+$	–	0.0314					
	$6_1^+ \to 8_1^+$	–	0.0155					
	$8_1^+ \to 10_1^+$	–	–					

4. Conclusions

In code NuSheIIX@MSU the f5pvh interaction have been performed by using model space f5p for $^{62-68}$Zn nuclei. The predicted low-lying levels (energies, spins, and parities), the reduced probabilities of $B(E2)\uparrow$ of transitions intrinsic quadrupole moments Q_0, and deformation parameters β_2 for even $^{62-68}$Zn nuclei have also been calculated. There is an agreement to some extent between the energy levels that have been calculated for the even-even $^{62-68}$Zn isotopes and the experimental values. The energy levels number has been confirmed and determined for which the angular momentum and parity are not well confirmed and specified empirically. There is an excellent agreement with some energy levels, reduced probabilities of $B(E2)\uparrow$ transitions, intrinsic quadrupole moments Q_0, and deformation parameters β_2 and the experimental values available for $^{62-68}$Zn isotopes. It has been concluded that the shell model configuration mixing in this region is very successful; also new values of energy levels, reduced transition probabilities are predicted in these studied calculations, these values have not been evaluated at the experimental results. This investigation increases the theoretical knowledge of all isotopes with respect to the energy levels and reduced transition probabilities. In this study, the results are very useful for compiling the nuclear data table [25, 26].

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РОЗРАХУНКИ ПАРАМЕТРА КВАДРУПОЛЬНОЇ ДЕФОРМАЦІЇ β_2 З НАВЕДЕНИХ ІМОВІРНОСТЕЙ ПЕРЕХОДУ *B*(*E*2)↑ ДЛЯ $0_1^+ \rightarrow 2_1^+$ ПЕРЕХОДІВ У ПАРНО-ПАРНИХ $^{62\text{-}68}$Zn ІЗОТОПАХ

Розраховано енергії збуджених рівнів, наведено імовірності переходу *B*(*E*2)↑, квадрупольні моменти та параметри деформації для ізотопів $^{62\text{-}68}$Zn з числом нейтронів N = 32, 34, 36 та 38. Для всіх станів ядер fp-оболонки застосовувався код NuShellX. Розрахунки по оболонковій моделі для ізотопів цинку проводились із частинками на орбітах $lp_{3/2}$, $0f_{5/2}$ та $lp_{1/2}$ за межами подвійно-магічного ядра ^{56}Ni. Використовуючи модельний простір f5p та f5pvh взаємодію, було отримано теоретичні результати, які було порівняно з наявними експериментальними даними. Значення енергій збудження, імовірностей переходу *B*(*E*2), квадрупольних моментів Q_0 та параметрів деформації β_2 знаходяться в повній згоді з експериментальними значеннями. Крім того, були визначені рівні енергій для кутових моментів та парностей, які були недостатньо встановлені та визначені експериментально. Було також передбачено деякі нові рівні енергії та ймовірності електричних переходів для ізотопів $^{62\text{-}68}$Zn, які раніше були відсутні в експериментальних даних.

Ключові слова: *B*(*E*2)↑, основні стани, код NuShellX, параметри деформації.

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