ЯДЕРНА ФІЗИКА
NUCLEAR PHYSICS

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# DETERMINATION OF THE NUCLEAR RADIUS PARAMETER USING THE γ-RAY SPECTROMETER

The nuclear radius parameter of carbon, aluminium, iron, copper, and zinc nuclei has been determined by using (n, γ)-reaction. The neutrons from the americium-beryllium source are made to interact with the water moderator to produce the γ-rays of 2.2 MeV through (n, γ)-reaction. The γ-radiation emitted from the water medium is measured with a scintillation detector coupled to 8k multi-channel analyzer. The neutrons from the americium-beryllium source are allowed to transmit through carbon, aluminium, iron, copper, and zinc elemental targets of various thicknesses, and transmitted neutrons are again allowed to interact with water moderator to produce 2.2 MeV γ-radiation. By measuring the yield of γ-radiation produced in water moderator by neutrons transmitted through elemental targets of different mass number values, the total neutron interaction cross-sections are determined. By knowing the total neutron interaction cross-sections and mass number of the target nuclei, the radius parameter has been determined.

*Keywords:* americium-beryllium neutron source, scintillation detector, neutron interaction cross-section, nuclear radius parameter, (n, γ)-reaction.

## 1. Introduction

Several researchers have already carried out experiments in nuclear and radiation physics using weak radioactive sources to determine various parameters such as rest mass energy of electrons, the binding energy of electrons, effective atomic number, and verification of Moseley’s law [1 - 8]. However, the measurement of the radius of a nucleus using a weak neutron source has been a challenging topic in nuclear physics. In the present investigation, the authors have shown that the nuclear radii of medium atomic number Z elements can be determined using a weak neutron source and low energy resolution γ-detector.

Atomic nuclei are composed of neutrons and protons, and they are bound together by a short-range nuclear attractive force. The spherical nuclei possess the least surface area and provide maximum nuclear force to bind the nucleons in the nucleus. This signifies that the spatial distribution of protons and neutrons is uniform throughout the nucleus. The radius R of such a nucleus is given by $R = R_o A^{1/3}$, where $R_o$ is the radius parameter. This indicates that R has a dependency on the mass number A, R is proportional to $A^{1/3}$. Kenneth Krane [9] has stated that the value of nuclear radius depends on the kind of experiment involved. The experiments such as high energy electron scattering, muonic X-rays, optical and X-ray isotope shifts, and energy differences of mirror nuclei would determine nuclear charge distribution, while Rutherford scattering, alpha-decay, and pionic X-rays and neutron scattering determine the distribution of nucleons. Several investigators have adopted various experimental as well as theoretical techniques to determine the radius parameter of the nuclei [10 - 18]. The neutron scattering experiment is one of the simplest and low-cost undergraduate experiments to measure the radius parameter as indicated by the reference [18]. In this experiment, the total neutron cross-section $\sigma_T$ for the interaction of incident neutrons with the various targets of mass number A is measured. The plot of $\sqrt{\sigma_T / 2\pi}$ versus $A^{1/3}$ gives a straight line and slope would give the radius parameter. In all these experiments, the neutrons are measured with a stilbene crystal or plastic scintillator coupled to a photomultiplier. Some of the graduate laboratories although carrying out γ-ray interaction experiments are unable to perform basic nuclear physics experiments such as the measurement of nuclear radius due to the lack of a neutron detector. In the present work, authors have shown for the first time that the nuclear radius parameter of medium Z nuclei can be determined by converting a neutron into γ-radiation through (n, γ)-reaction and by measuring the γ-radiation with available scintillation γ-ray spectrometer.

## 2. Theory

When the neutrons are incident on a target, some are scattered, some are absorbed, and some are transmitted through the target without interaction with the target nuclei. The absorption and scattering of neutrons by the nucleus depend on the de-Broglie wavelength λ of the incident neutron. When λ is small compared to the size of a nucleus, a large number of neutrons are not scattered by the nucleus. On the other hand, if λ is equal to or greater than the size of a nucleus, most of the neutrons would undergo collision with the nucleus. Consequently, the incident neutrons are either scattered or absorbed. The probability of scattering or absorption can be expressed in terms of the cross-section. The cross-section is the ratio of the number of scattering or absorption per unit time per nucleus to the number of incident particles per unit time per unit area. Hence the total cross-section $\sigma_T$ is the sum of the scattering section $\sigma_s$ and absorption section $\sigma_a$. The $\sigma_T$ can be determined by adopting the neutron transmission method. Several investigators have measured the total neutron cross-sections to determine the nuclear radius parameter. However, in the present investigation, the incident and transmitted neutrons are converted into 2.2 MeV γ-radiations by the interaction of neutrons with the water medium [19]. The energy and yield of γ-radiation are measured with NaI(Tl) γ-ray spectrometer. Therefore, the yield of γ-radiation is directly connected to the yield of an incident neutron [19]. Let $I_o$ be the yield of γ-radiations detected by the scintillation detector due to the interaction of $N_o$ number of neutrons with the water medium. Let I be the number of γ-radiation due to the interaction of N number of neutrons with water medium after transmitted through an elemental target of thickness t. The yield of γ-photon I is given by

$$I = I_o e^{-\mu t}, \tag{1}$$

where μ is the linear attenuation coefficient of neutrons in the given target of thickness t. Then, the $\sigma_T$ can be expressed as

$$\sigma_T = \frac{\mu}{n}, \tag{2}$$

where n is the number of nuclei per $cm^3$ of the target and equation (1) can be written as

$$I = I_o e^{-\sigma_T nt}, \tag{3}$$

$$\frac{I}{I_o} = T = e^{-\sigma_T nt}, \tag{4}$$

$$\ln T = -\sigma_T nt, \tag{5}$$

$$\sigma_T = -\frac{\ln T}{nt}, \tag{6}$$

where nt is the number of nuclei per $cm^2$. The total cross-section can be determined by knowing $I_o$ and I [20]. According to the optical model of the nucleus, the total cross-section can be given by

$$\sigma_T = \sigma_s + \sigma_a, \tag{7}$$

where $\sigma_s$ is the scattering cross-section and $\sigma_a$ is the absorption (reaction) cross-section. The total cross-section can be written as mentioned in Ref. [21].

$$\sigma_T \approx \pi\left(R + \lambda\!\!\!^{-}\right)^2 + \pi\left(R + \lambda\!\!\!^{-}\right)^2, \tag{8}$$

$$\sigma_T \approx 2\pi\left(R + \lambda\!\!\!^{-}\right)^2, \tag{9}$$

where the radius of nucleus $R = R_o A^{1/3}$, A is a mass number of the scattering sample and $\lambda\!\!\!^{-}$ is the reduced de-Broglie wavelength of an incident neutron. The $\sigma_T$ can be written as

$$\sqrt{\sigma_T / 2\pi} = R_o A^{1/3} + \lambda\!\!\!^{-}. \tag{10}$$

From the plot of $\sqrt{\sigma_T / 2\pi}$ versus $A^{1/3}$, the radius parameter $R_o$ can be determined.

## 3. Experimental Details and Data analysis

The experimental arrangement used in the present investigation is shown in Fig. 1. It consists of an americium-beryllium neutron source that is shielded into the cylindrical paraffin container of 20 inch height and 10 inch in diameter, a lead absorber of ~2 cm thick on the paraffin container, elemental target samples of 4 cm × 4 cm, water moderator of 5 cm ×5 cm and 2” × 2” NaI(Tl) γ-ray spectrometer coupled to 8k multichannel analyzer.

The transmitted neutrons through the lead and elemental targets were trimmed with collimators $C_1$ and $C_2$ to have a good geometrical interaction with the water moderator. The $C_3$ collimator was used to trim the obtained 2.2 MeV gamma from the water moderator essentially due to p(n, γ)d-reaction. The γ-ray spectrometer was calibrated using energy lines of $^{137}Cs$, $^{60}Co$, $^{22}Na$, $^{54}Mn$, $^{133}Ba$ radioactive nuclides. The calibration constant was found to be $(1.70 \pm 0.03)\cdot 10^{-3}$ MeV/channel with a linear fit. The americium-beryllium neutron source has a half-life of 432.2 yr and emits about $10^5$ neutrons per unit area per second with an average energy of 4.2 MeV [22]. Along with neutrons, γ-rays of 59.5 keV and 4.4 MeV are also emitted from the nuclear de-excitation of $^{241}Am$ and $^{12}C$, respectively [23].

In order to obtain the neutrons which are almost free from γ-radiations, a lead shield of thickness ~2 cm was placed at the exit window of the paraffin container. The lead has been used for attenuation of

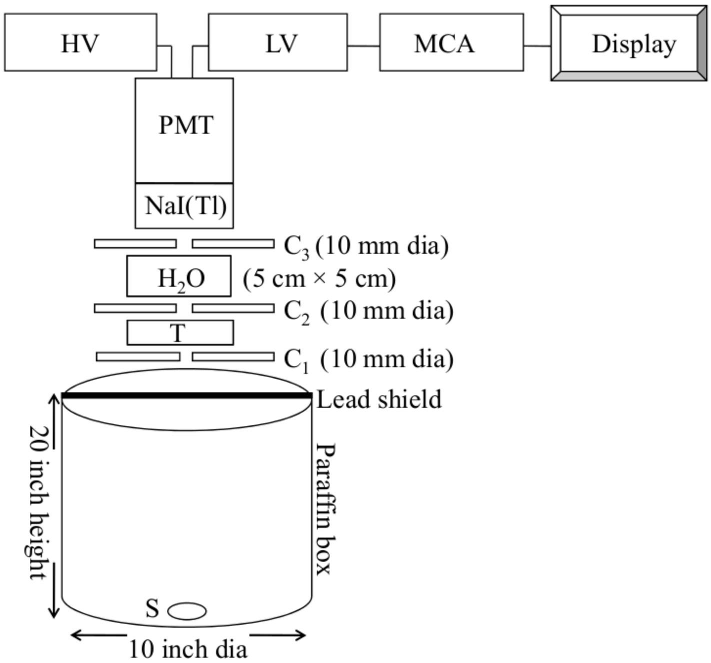


Fig. 1. A schematic experimental arrangement to determine the nuclear radius parameter of medium atomic number elements using a weak neutron source. S - neutron source; T - targets; $H_2O$ - water medium; $C_1$, $C_2$, and $C_3$ - collimators; NaI(Tl) - scintillator crystal; PMT - photo-multiplier tube; HV - high voltage terminal; LV - low voltage terminal; MCA - multichannel analyzer.

γ-radiation and for transmission of neutrons; one can expect only transmitted neutrons with maximum reduction in the gamma from source as well as background radiation. The transmitted neutrons from the lead absorber are allowed to interact with the water moderator to produce a 2.2 MeV γ-photon through (n, γ)-reaction. The resolution of the detector at 2.2 MeV γ-radiations was found to be ~0.250 MeV. The area under the peak would give the yield of the 2.2 MeV γ-radiations which are essentially due to the interaction of neutrons with the water moderator. The neutrons transmitted through the lead absorber are allowed to pass through the elemental targets of carbon, aluminium, iron, copper, and zinc. The transmitted neutrons through these foils interact with the water moderator to produce 2.2 MeV γ-radiations.

The yield of the γ-radiation is measured for the various thicknesses of the given elemental foils. The data were acquired for 4000 s at each target sample throughout the experiment. The typical γ-spectra produced by neutrons in a water medium after passing through various thicknesses of copper elemental target is shown in Fig. 2. It is noticed from the Figure that the yield of γ-radiation decreases with an increase in the foil thickness, indicating the yield of neutron decreases with increasing the thickness of the target. By knowing the yield of γ-photon of $I_o$ and I, the total cross-sections were determined for various Z values, and they are given in Table 1.

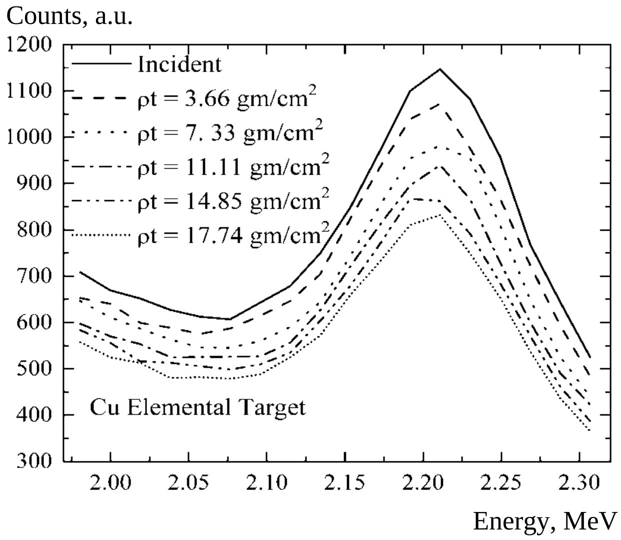


Fig. 2. The typical γ-ray spectrum of (n, γ)-reaction for copper target at various mass thicknesses. ρ - the density of the target, $gm/cm^3$; t - the thickness of the target, cm.

*Table 1.* **The total neutron interaction cross-section ($\sigma_T \cdot 10^{-24}$ cm$^2$) for elemental targets**

| Target | $\sigma_T \cdot 10^{-24}$ cm$^2$ |
|---|---|
| Carbon | 0.848 ± 0.055 |
| Aluminium | 1.237 ± 0.139 |
| Iron | 1.780 ± 0.279 |
| Copper | 1.851 ± 0.263 |
| Zinc | 2.055 ± 0.684 |

A typical plot of $\ln(I/I_o)$ at various nt for copper elemental targets is shown in Fig. 3. The plot of $\sqrt{\sigma_T / 2\pi}$ versus $A^{1/3}$ gives a straight line; slope gives the radius parameter. The slope obtained from Fig. 4 in the present investigation is 1.10 ± 0.068 fm. With this, the radius of carbon, aluminium, iron, copper, and zinc nuclei was determined using the relation $R = R_o A^{1/3}$.

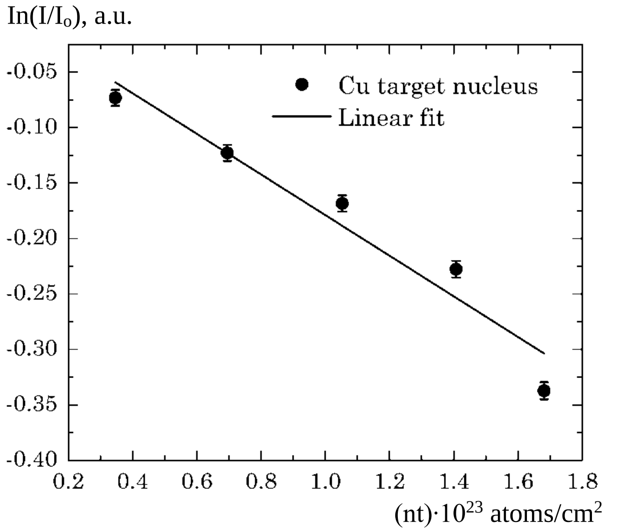


Fig. 3. The experimental points of transmission coefficient $\ln(I/I_o)$ as a function of the number of target atoms per unit area nt for the copper target are fitted with a linear equation (linear fit).

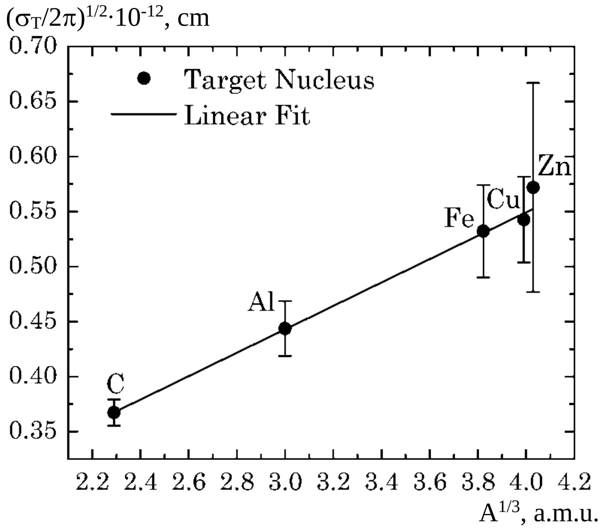


Fig. 4. The experimentally obtained total neutron interaction cross-section divided by 2π under the root is plotted as a function of $A^{1/3}$ for elemental targets and fitted with a linear equation (linear fit).

*Table 2.* **Comparison of the radii of target nuclei determined using the present experiment technique and values observed from earlier studies using different techniques**

| Target nucleus | Radii, fm [This work] | Radii, fm [Ref. 13] | Radii, fm [Ref. 24] |
|---|---|---|---|
| Carbon | 2.52 ± 0.16 | 2.450 | 2.817 |
| Aluminium | 3.30 ± 0.20 | 3.209 | 3.689 |
| Iron | 4.21 ± 0.26 | 4.089 | 4.701 |
| Copper | 4.39 ± 0.27 | 4.269 | 4.908 |
| Zinc | 4.44 ± 0.27 | 4.311 | 4.956 |

A comparison of the radii values determined by this experiment with others is given in Table 2 [13, 24]. The close agreement between our experimental values and the others indicates that the present technique can be adopted in graduate laboratories that do not have the neutron detector.

## 4. Conclusions

The radii of medium Z nuclei have been determined by using (n, γ)-reaction. The γ-radiations of 2.2 MeV are produced by the interaction of neutrons with the water medium. By adopting the transmission of neutrons in medium Z elements, the total cross-section is determined. From the total cross-section, the radius parameter was determined and found to be 1.10 ± 0.068 fm. Using this radius parameter, the radii of the medium Z elements have been determined using the relation, $R = R_o A^{1/3}$. As it is a novel laboratory experiment for graduate and undergraduate students, the error in the measured values of the radius is not so important. However, the error in the measured values is about 6 to 7 %. The experimentally determined values have been compared with theoretical and experimental values. The good agreement indicates that the present method can be adopted by graduate laboratories using a scintillation γ-ray spectrometer.

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## ВИЗНАЧЕННЯ РАДІУСА ЯДЕР ЗА ДОПОМОГОЮ γ-СПЕКТРОМЕТРА


Радіуси ядер вуглецю, алюмінію, заліза, міді та цинку визначено за допомогою (n, γ)-реакції. Нейтрони з америцій-берилієвого джерела взаємодіють з водним сповільнювачем, що призводить до випромінення γ-квантів з енергією 2,2 МеВ через (n, γ)-реакцію. Це γ-випромінювання вимірюється сцинтиляційним детектором, підключеним до багатоканального аналізатора 8k. Нейтрони з америцій-берилієвого джерела пропускались через мішені з вуглецю, алюмінію, заліза, міді та цинку різної товщини, і потім також взаємодіяли з водним сповільнювачем для отримання 2,2 МеВ γ-випромінювання. При вимірюванні виходу γ-квантів визначались повні перерізи взаємодії нейтронів. Із величини повного перерізу визначався параметр, що входить в опис залежності радіуса ядер від атомної маси.



*Ключові слова*: америцій-берилієве джерело нейтронів, сцинтиляційний детектор, поперечний переріз взаємодії нейтронів, радіус ядра, (n, γ)-реакція.

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