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ELASTIC AND INELASTIC FORM FACTORS OF THE ^{10}B NUCLEI WITH THE LARGE-BASIS SHELL MODEL

In this study, inelastic and elastic form factors for the low-lying excited states of ^{10}B nucleus were calculated utilizing the nuclear shell model theory. We employed a large-basis psd model space with psdmwk interaction and the harmonic oscillator potential in the form factors calculation. The calculated results with the effective charge are in acceptable agreement with experimental results.

Keywords: electron scattering, p-shell, core polarization, transverse and longitudinal form factor.

1. Introduction

In order to obtain important facts about the characteristics of atomic nuclei and nuclear structure at high energy, electron scattering is the main method used. The electron scattering from a target nucleus can occur in two ways: elastic and inelastic scattering [1]. The shell model is one of the important nuclear models in studying the structure and physical properties of the nucleus [2]. Studying the nuclear structure through electron scattering is important because the nuclear matrix of the element depends on the momentum transfer that gives information about the structure of both the ground and the excited density states [3]. In the p-shell model space, the nucleons are distributed over the $1p_{3/2}$ $1p_{1/2}$ only. This model has unsuccessfully obtained form factors and transition rate in comparison with experimental data unless modified by insertion of the core-polarization effect, which gives better results consistent with the experimental data. The expanded shell model has included two shells (1p, 1d-2s) [4].

The magnetic form factors of the p-shell nuclei (^{6}Li, ^{10}Be, ^{11}B, ^{14}N, ^{15}N) were calculated by Booten and Van Hees [5]. Their shell model calculation using the harmonic-oscillator inclusion of the 1p-shell and spsdpf-shell truncated up to (0 + 2) ħω enhanced the form factors convergence with experimental results.

Cichocki et al. [6] measured the transverse and longitudinal form factors of ^{10}B and compared their result with the calculated 1p-shell model and large-basis calculation, which includes 1s, 2s1d, and 2p1f configurations. They found that only 10 % improvements were realized and found that including higher excited configuration employing core-polarization calculation was essential to remove the remaining shortfall.

Radhi et al. [7] studied the quadrupole Coulomb form factors (C2) transitions for p-shell nuclei, including the core-polarization effects excited up to 6ħω. They have verified that the effects of core-polarization are fundamental in both the momentum transfer and transition strengths and their results were in good agreement with no adjustable parameters.

Radhi et al. [8] studied the effective charges and quadrupole moments for B (A = 8, 10, 11, 12, 13, 14, 15) and Li (A = 7, 8, 9, 11) in p and large-basis model spaces where particles excited to higher orbits at 6ħω were included. Their calculated results agree very well with the experimental data.

Salman et al. [9] studied the longitudinal form factor in addition to the transverse one for ^{7}Li and ^{10}B nuclei. Their calculation included 1p-1h excitation up to 12ħω and is in a good accordance of the longitudinal form factor for ^{7}Li and also transverse form factor compared with 6ħω energies. Recently, Salman et al. [10] studied the large-basis calculation of the ^{6}Li nucleus with Skyrme potential. The even parity energy level and multipolar form factor large-basis calculations agree well with the experimental data.

In the present work, longitudinal and transverse form factors for ^{10}B nuclei were calculated with large-basis psd model space using the shell model Nushell @MSU code [11]. In the psd model, the 2s-1d shell is added to the p-shell account to get better results. Both form factor calculations utilize the Harmonic Oscillator potential for the single-particle wave function.

2. Theory

The longitudinal (L) and transverse (T) form factors are defined by [12]:

$$\left|F_J^L(q)\right|^2 = \frac{4\pi}{z^2}\frac{1}{2J_i+1}\sum_{J\geq 0}\left|\left\langle J_f\ M_{Jf}\left|\hat{T}_J^{Coul}(q)\right|J_I\ M_{Ji}\right\rangle\right|^2, \quad (1)$$

$$\left|F_J^T(q)\right|^2 = \frac{4\pi}{z^2}\frac{1}{2J_i+1}\Sigma\left|\left\langle J_f\ M_{Jf}\left|\hat{T}_J^{el}(q)\right|J_I\ M_{Ji}\right\rangle\right|^2 + \left|\left\langle J_f\ M_{Jf}\left|\hat{T}_J^{mag}(q)\right|J_I M_{Ji}\right\rangle\right|^2. \quad (2)$$

The multipole operator is defined by [12]:

$$\hat{T}_J^{Coul}(q) = \int \overrightarrow{dr} j_J(qr) S_J^M(\Omega_r)\hat{\rho}(\bar{r}), \quad (3)$$

$$\hat{T}_J^{el}(q) = \frac{1}{q}\int \overrightarrow{dr}\left\{\vec{\nabla}\cdot\left[j_J(qr)S_{JJ_1}^M(\Omega_r)\right]\right\}\cdot\hat{j}(\bar{r}), \quad (4)$$

$$\hat{T}_J^{mag}(q) = \int \overrightarrow{dr}\left[j_J(qr)S_{JJ_1}^M(\Omega_r)\right]\cdot\hat{j}(\bar{r}). \quad (5)$$

The form factor can be written as [12]:

$$\left|F_{L.T}^{\Lambda}(q)\right|^2 = \frac{4\pi}{z^2(2J+1)}\left|\sum_{T=0.1}(-1)^{T_f-T_Z}\begin{pmatrix}T_f & T & T_i\\ -T_Z & 0 & T_Z\end{pmatrix}\left\langle J_f T_f\left\|| T^{\Lambda}(q)\|\right| J_i T_i\right\rangle\right|^2\cdot\left|F_{f.s}F_{c.m}\right|^2, \quad (6)$$

where F_{cm} is the center of the mass correction factor, $F_{f.s}$ size correction, Λ selects the longitudinal or transverse form factors. The relation between these OBDM and the p/n OBDM is [13]:

$$\mathrm{OBDM}(p/n) = (-1)^{\tau f-\tau Z}\begin{pmatrix}T_f & 0 & T_i\\ -T_Z & 0 & T_Z\end{pmatrix}\sqrt{2}\,\frac{\mathrm{OBDM}(\Delta T=0)}{2},$$

$$(+/-)\tau Z(-1)^{Jf-JZ}\begin{pmatrix}T_f & 1 & T_i\\ -T_Z & 0 & T_Z\end{pmatrix}\sqrt{6}\,\frac{\mathrm{OBDM}(\Delta T=1)}{2}. \quad (7)$$

The multiparticle transition amplitudes are defined as:

$$\mathrm{OBDM}(I, j, L, \Delta T, j_1, j_2) = \frac{\left\langle J_f T_f\left\|\left[\alpha_{j_1 j_3}^{\dagger}\otimes\tilde{\alpha}_{j_2 j_3}\right]^{r}\right\|J_i T_i\right\rangle\sqrt{2J_j+1}}{\sqrt{2L+1}\sqrt{2\Delta T+1}}, \quad (8)$$

where $j_3 = 1/2$ for neutron and $j_3 = -1/2$ for a proton. In the single nucleon state (j_1 j_3) the corrosion $(\alpha^{\dagger})$ generate a neutron or proton, from the single nucleon state (j_2 j_3) the annihilation $(\tilde{\alpha})$ remove a neutron or proton.

3. Results and discussion

The calculated longitudinal and transverse form factors for the ^{10}B nucleus used the psd model space. The chosen oscillator length parameter is b = 1.76 fm^{-1}.

3.1. Longitudinal form factors

The square form factors ($|F(q)|^2$) versus the momentum transfer (q) in the ^{10}B nucleus for all figures are calculated by employing the harmonic-oscillator potential. The calculated results of the psd model (solid line) are compared with p-shell (dashed line) and spsdpf (dashed-dot line) truncation at 2ħω models. In Fig. 1 the longitudinal inelastic (C2) form factors are calculated at E_x = 0.718 MeV. The results using the psd model space with defaults effective charge give a good description of the experimental data best from p and spsdpf models except for the region $1 \leq q \leq 2$ fm^{-1}, while the results of p-shell and spsdpf-shell underestimated experimental data at $q \leq 2.5$ fm^{-1}.

The psd model calculation longitudinal inelastic (C2) form factors at E_x = 2.154 MeV with default effective charge (0.35, 0.35) in Fig. 2 overestimates the experimental data shape at $q \leq 2$ fm^{-1}. But it was able to produce the form factors that match the experimental data shape with high transfer momentum.

The calculated longitudinal inelastic (C2) form factors for the J^{π} T = 1^{+} 0(Ex = 3.587 MeV) state in ^{10}B are displayed in Fig. 3. The calculated results using the psd model agree well with experimental data at all momentum transfer values, especially at higher momentum transfer values.

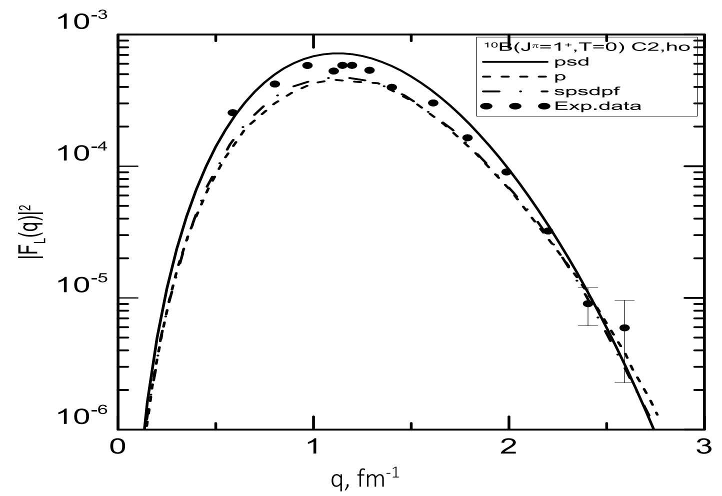

Fig. 1. The inelastic C2 form factor of the 1^+ (0.718 MeV) state for ^{10}B
was calculated using psd, p, and spsdpf model space. The data are extracted from Ref. [6].

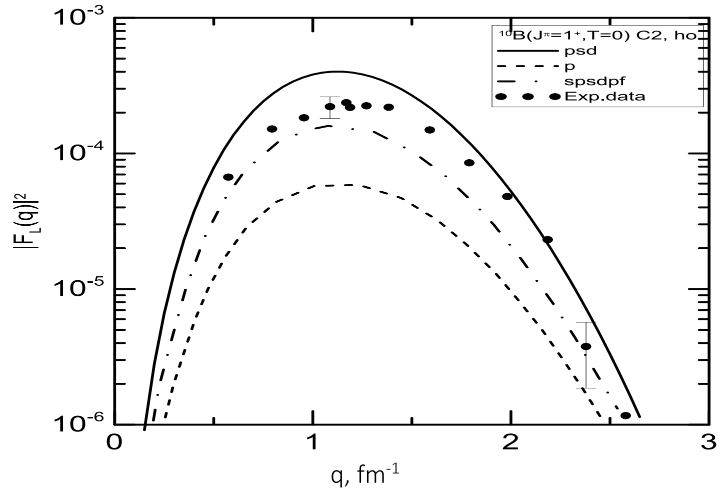

Fig. 2. The inelastic C2 form factors of the 1^+ (2.154 MeV) state for ^{10}B
were calculated using psd, p, and spsdpf models space. The data are extracted from Ref. [6].

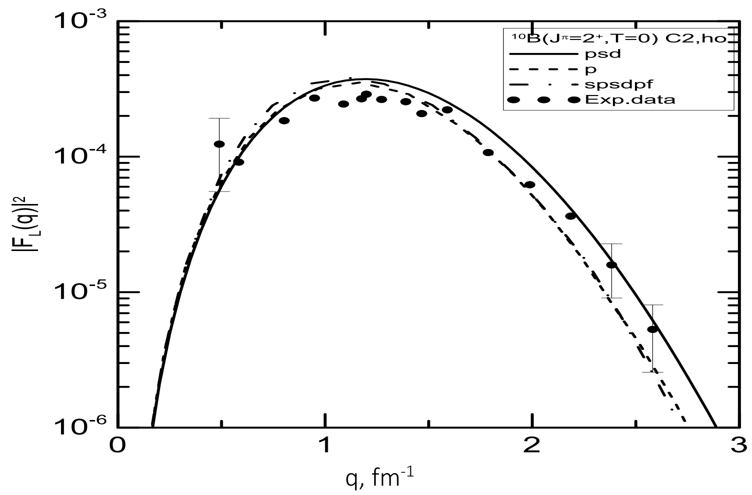

Fig. 3. The inelastic C2 form factors of the 2^+ (3.587 MeV) state for ^{10}B
were calculated using psd, p, and spsdpf models space. The data are extracted from Ref. [6].

Fig. 4 shows the calculated longitudinal inelastic C2 form factor for the $J^\pi T = 4^+ 0$ at $E_x = 6.025$ MeV state in ^{10}B using the psd model space with an effective charges value equal to (0.8, 0.4) for proton and neutron, respectively. The calculated results are in a good agreement and are closer than other results of [6] to experimental data.

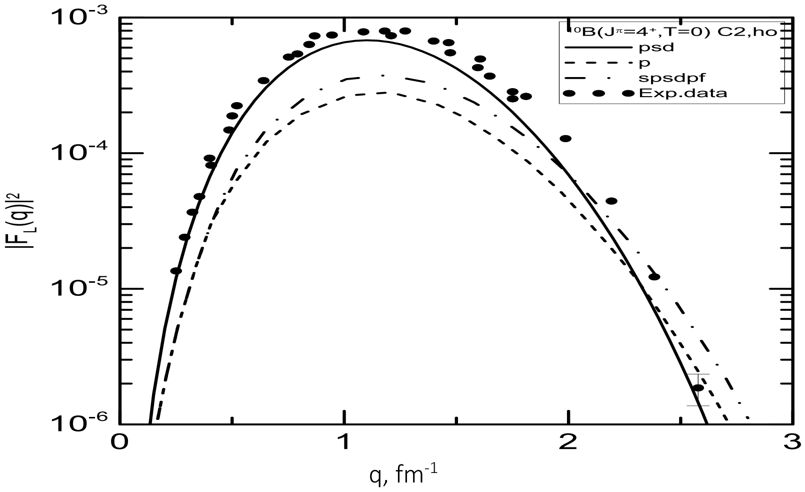

Fig. 4. The inelastic C2 form factors of the 4^+ (6.026 MeV) state for ^{10}B were calculated using psd, p, and spsdpf models space. The data are extracted from Ref. [6].

3.2. Transverse form factors

Fig. 5 shows the total transverse (M1 + M3) form factors of the (J^{π} T = 3^+ 0) state in ^{10}B calculated at E_x = 0.00 MeV. The results of the calculations with the use of psd model space at default effective charge agree well with experimental data at q ≥ 1.9 fm^{-1}. Where's, that gives compatible results at higher momentum transfer. The psd calculation gives the minimum deviation at q = 2 fm^{-1}.

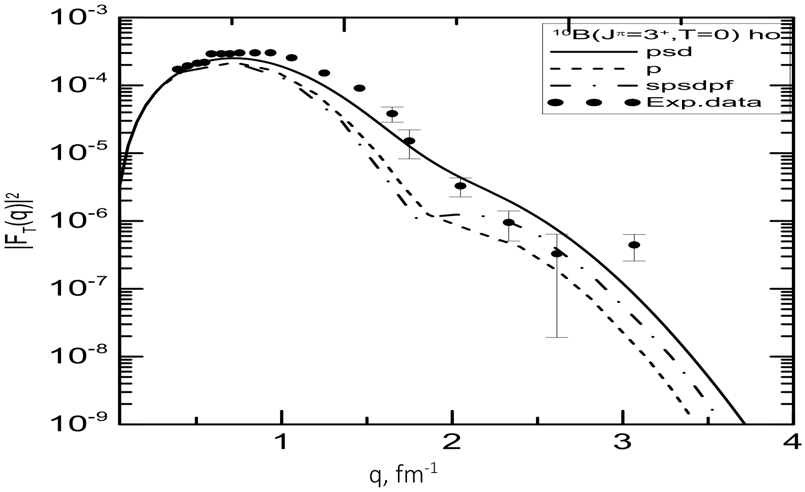

Fig. 5. The transverse elastic (M1 + M3) form factors for the 3^+ (0.00 MeV) state in ^{10}B were calculated using psd, p, and spsdpf models space. The data are extracted from Ref. [5].

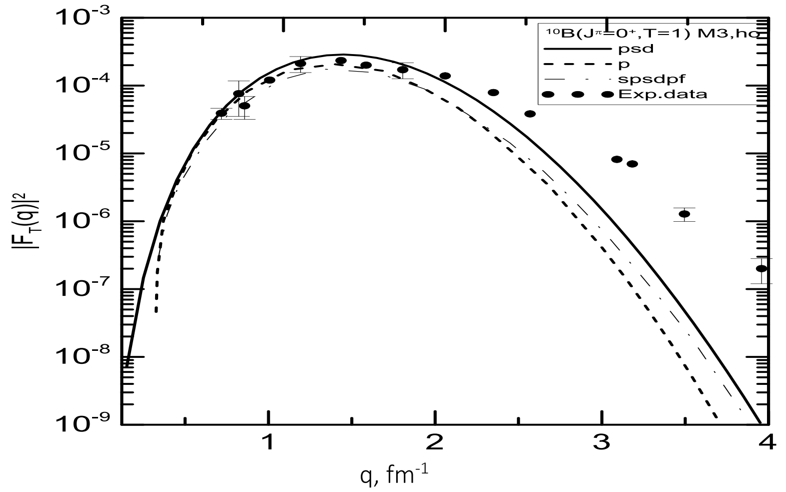

Fig. 6. The transverse inelastic M3 form factors for the 0^+ (1.74 MeV) state in ^{10}B were calculated using psd, p, and spsdpf models space. The data are extracted from Ref. [5].

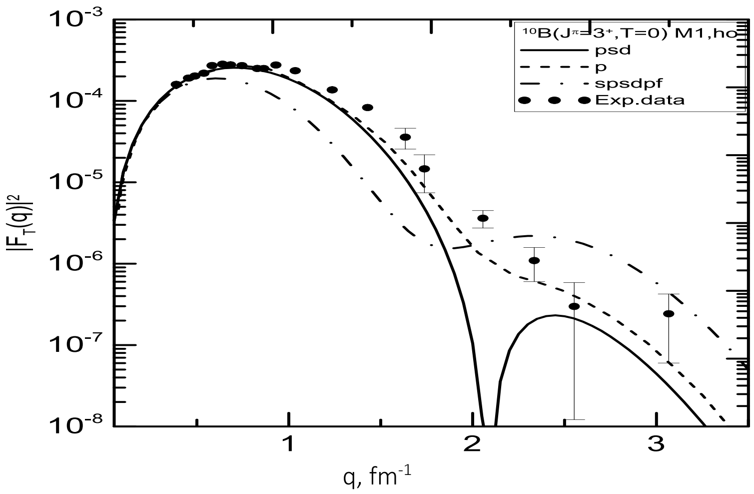

Fig. 7. The transverse inelastic M1 form factors for the 3$^+$ (0.00 MeV) state in ^{10}B were calculated using psd, p, and spsdpf models space. The data are extracted from Ref. [5].

Fig. 6 shows the calculated inelastic transverse (M3) form factors for the (J^{π} T = 0$^+$ 1) at (E_x = 1.74 MeV) state in ^{10}B compared with data from Ref [5]. The calculated results are in good agreements with the experimental data at lower values of momentum transfer q ≤ 2 fm^{-1} but deviate for higher values of momentum transfer. Fig. 7 displays the pure M1 transition form factors calculated for (J^{π} T = 0$^+$ 1) state at E_x = 0.00 MeV in the ^{10}B nucleus. At the higher values of q, the calculated form factor underestimates the experimental values while giving consistent results at lower momentum transfer.

4. Conclusion

The calculated results of longitudinal and transverse form factors using the psd model space with psdmwk interaction give good agreement compared with experimental data. The large-base psd model seems to be enough to obtain the best description and is closer to experimental results than the two p and spsdpf models-space.

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ПРУЖНІ ТА НЕПРУЖНІ ФОРМ-ФАКТОРИ ЯДРА ^{10}B В ОБОЛОНКОВІЙ МОДЕЛІ З ВЕЛИКИМ БАЗИСОМ

У цій роботі були розраховані непружні та пружні форм-фактори для збуджених станів ядра ^{10}B з низькими енергіями в ядерній оболонковій моделі. Для розрахунку форм-факторів було використано модельний простір psd з великим базисом із взаємодією psdmwk і потенціалом гармонічного осцилятора. Отримані результати з ефективним зарядом знаходяться в прийнятному узгодженні з експериментальними даними.

Ключові слова: розсіяння електронів, p-оболонка, поляризація ядра, поперечний і поздовжній форм-фактор.

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