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STUDY OF SPECTROSCOPIC PROPERTIES OF SOME NUCLEI PARTICIPATING IN (μ^-, e^-) LEPTON FLAVOR VIOLATION PROCESS

Lepton flavor violation (LFV) is a clear sign of new physics beyond the standard model. A prominent process concerning LFV is $\mu^- \to e^-$ conversion in a muonic atom. In the present work, we have investigated the spectroscopic properties of three nuclei namely ^{24}Mg, ^{32}S, and ^{44}Ca which participate in this $\mu^- \to e^-$ lepton flavor violating process. We have used USD interaction for *sd* shell nuclei namely ^{24}Mg and ^{32}S and Z20 Bonn interaction for *pf* shell nucleus ^{44}Ca, to calculate these properties.

Keywords: lepton flavor violation, muon to electron conversion, spectroscopic properties, muon sources.

1. Introduction

The observation of neutrino oscillations implies that flavor violation also occurs among the charged leptons, though the neutrino-mediated contribution to charged lepton flavor violation (CLFV) is unobservable, suppressed by the small neutrino mass. Consequently, observation of CLFV would be evidence of additional new physics [1 - 9]. One of the most sensitive tests of CLFV is $\mu^- \to e^-$ conversion, in which muons are stopped in a target, captured into the 1s Coulomb orbits of the target nuclei, and converted into mono-energetic electrons, unaccompanied by any other final-state leptons. The final state of the nucleus could be either the ground state or the excited state. In general, the transition process to the ground state, which is called coherent capture, is dominant. The rate of the coherent capture process over noncoherent ones is enhanced by a factor approximately equal to the number of nucleons in the nucleus since all of the nucleons participate in the process. The possible contributions to $\mu^- \to e^-$ conversion in a muonic atom can be grouped into two parts, which are the photonic contribution and the nonphotonic contribution. The study of the photonic contribution was initiated by [10]. The nonphotonic contribution was studied later, for instance by [11]. In order to study these photonic and nonphotonic contributions, the knowledge of concerned nuclear transition matrix elements (NTMEs) is required and these NTMEs can be calculated only when one has a set of reliable wave functions. Therefore, in the present work, we have calculated the spectroscopic properties of some nuclei namely ^{24}Mg, ^{32}S, and ^{44}Ca to judge the validity of the wave functions. These wave functions can then be used to calculate the required NTMEs. The present work is organized as follows. In Section 2, we outline the framework to calculate the spectroscopic properties namely yrast energies, quadrupole moment [$Q(2^+)$], magnetic moment (μ), and reduced transition probabilities [B(E2)] for the above nuclei. In Section 3, we have given the numerical results and discussed them. Finally, Section 4 is devoted to conclusions.

2. Shell model space and spectroscopic properties

In the present work, we have used the ANTOINE shell model code [12, 13] to calculate the spectroscopic properties. For *sd* shell ^{24}Mg, and ^{32}S nuclei the model space consists of three single particle orbits namely $0d_{3/2}$, $0d_{5/2}$, and $1s_{1/2}$ with single particle energies taken as 1.6465, –3.9478, and –3.1635 MeV, respectively, assuming ^{16}O nucleus as an inert core. Also, for *pf* shell nucleus ^{44}Ca, the model space has four single particle orbits namely $0f_{7/2}$, $1p_{3/2}$, $0f_{5/2}$, and $1p_{1/2}$ with single particle energies as 0.0, 2.0, 6.5, and 4.0 MeV, respectively, with ^{40}Ca nucleus assumed to be an inert core. The interactions used are USD for ^{24}Mg and ^{32}S nuclei while Z20 Bonn interaction is used for ^{44}Ca nuclei [14]. Using these single-particle energies and model space, we have calculated the energies of 2^+, 4^+, 6^+, and 8^+ states of the above nuclei and compared them with the available experimental data. The calculated values are shown in Table 1 and energy states are plotted in the Figure. Similarly, we have calculated reduced transition probabilities [B(E2)] for $0^+ \to 2^+$ transition and quadrupole moments [$Q(2^+)$] for 2^+ state and calculated values are given in Table 2 along with the available experimental data.

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The magnetic moment operator used in the present calculation is

$$\mu = g_s s + g_l l, \tag{1}$$

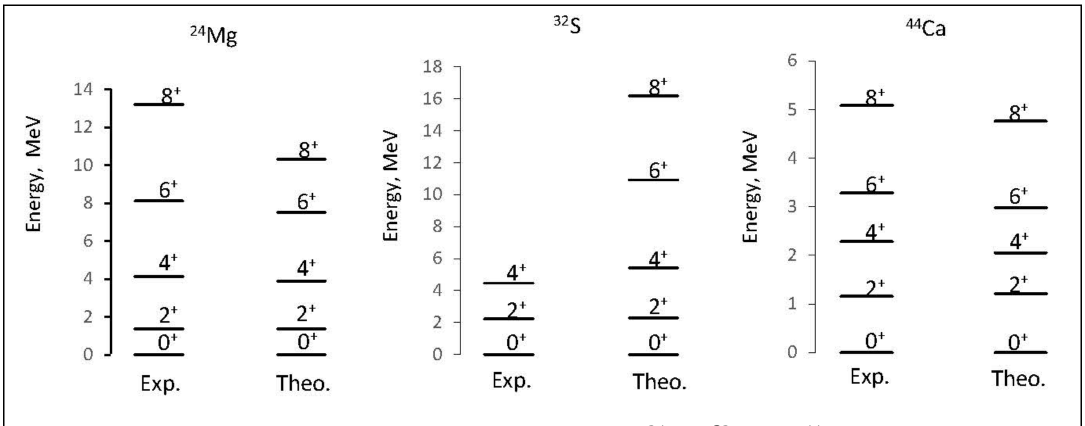
where g_s and g_l are the spins and the orbital g factors, respectively. By using the free-nucleon g factors $g_s = 5.586$, $g_l = 1$, for protons and $g_s = -3.826$, $g_l = 0$ for neutrons, the agreement between the calculated value (μ_{fth}) and experiment (μ_{exp}) appears to be reasonable. However, there are small but systematic deviations from the experimental values. Such deviations disappear almost when we introduce effective spin g factors, $g(\text{eff.}) = 0.7g(\text{free})$. Here, the "quenching" factor $q_s = 0.7$ is determined via least squares fit to the experimental data [14]. The magnetic moment values taking both sets of g factors are calculated and shown in Table 3.

3. Results and discussion

In Table 1, we have given the calculated and available experimental [15] energy eigenvalues of 2^+, 4^+, 6^+, and 8^+ states of nuclei.

Table 1. **Experimental and theoretical energy levels of ^{24}Mg, ^{32}S, and ^{44}Ca nuclei**

Energy levels	Nuclei					
	^{24}Mg		^{32}S		^{44}Ca	
	Energy, MeV		Energy, MeV		Energy, MeV	
	Exp.	Theo.	Exp.	Theo.	Exp.	Theo.
0^+	0.0	0.0	0.0	0.0	0.0	0.0
2^+	1.369	1.362	2.230	2.278	1.157	1.209
4^+	4.123	3.894	4.459	5.409	2.283	2.050
6^+	8.113	7.507	–	10.916	3.285	2.979
8^+	13.20	10.304	–	16.173	5.086	4.762



Experimental and theoretical energy levels of ^{24}Mg, ^{32}S, and ^{44}Ca nuclei.

From Table 1, it is observed that calculated values are in good agreement with the experimental data. We have also sketched the energy levels in the Figure. The experimental value of only 2^+ and 4^+ states of ^{32}S nuclei is available. The 2^+ states of ^{24}Mg and ^{32}S nuclei are in a good match while 2^+ state of ^{44}Ca is slightly off from the experimental value.

In Table 2 we have shown the results for reduced transition probabilities for $0^+ \rightarrow 2^+$ transition for two sets of proton and neutron effective charges i.e. (0.5, 1.5) and (1.1, 1.5) and compared them with available experimental data [16].

Table 2. **Reduced transition probabilities B(E2) for ^{24}Mg, ^{32}S, and ^{44}Ca nuclei**

Nuclei	B(E2: $0^+ \rightarrow 2^+$), e^2b^2		
	Exp.	Theo.	
		$(e_p, e_n) = (0.5, 1.5)$	$(e_p, e_n) = (1.1, 1.5)$
^{24}Mg	0.043	0.041	0.069
^{32}S	0.030	0.022	0.036
^{44}Ca	0.047	0.007	0.037

It is seen that calculated values are in close agreement with the experimental ones for effective charges 1.1 and 1.5 for proton and neutron respectively in the case of ^{32}S and ^{44}Ca nuclei. For ^{24}Mg, results match for the set e_p, e_n = 0.5, 1.5. In Table 3, we have presented the results for quadrupole moments [Q(2^+)] for 2^+ state and magnetic moments (μ) for considered nuclei and compared the results with experimental data [17].

Table 3. **Experimental and theoretical values of quadrupole moments (Q) of 2^+ state and magnetic moments for ^{24}Mg, ^{32}S, and ^{44}Ca nuclei**

Nuclei	J^π	Quadrupole moment, eb			Magnetic moment, μ_N		
		Q_{exp}	Q_{th1}	Q_{th2}	μ_{exp}	μ^{th}_{free}	μ^{th}_{eff}
^{24}Mg	2^+	–0.07	–0.18	–0.24	+1.02	+1.06	+1.02
^{32}S	2^+	–0.18	–0.14	–0.18	+0.90	+1.30	+1.18
^{44}Ca	2^+	–0.14	–0.07	–0.15	–0.60	–0.71	–0.49

The calculated quadrupole moments are shown in columns 4 and 5, written as Q_{th1} and Q_{th2} for two sets of effective charges (0.5, 1.5) and (1.1, 1.5) for proton and neutron, respectively while column 3 shows experimental values. The values of Q are calculated for 2^+ states. The Q_{th2} values show good agreement with the experimental ones for ^{32}S and ^{44}Ca nuclei, while the calculated Q value is slightly off from the experimental one in the case of ^{24}Mg. The magnetic moments are shown in columns 7 and 8 as μ^{th}_{free} and μ^{th}_{eff}, respectively. Experimental values of magnetic moments are shown in Column 6. The μ^{th}_{free} values are calculated taking free nucleon factors as $g_s = 5.586$, $g_l = 1$ for protons and $g_s = -3.826$, $g_l = 0$ for neutrons while μ^{th}_{eff} value is calculated by taking $g(\text{eff.})_s = 0.7\ g(\text{free})_s$. It is observed that the theoretical value of magnetic moment matches with the experimental one in case of ^{24}Mg for μ^{th}_{eff} while the two values are in close agreement for ^{44}Ca nuclei using free nucleon factors.

4. Conclusion

We have calculated the spectroscopic properties of ^{24}Mg, ^{32}S, and ^{44}Ca nuclei which participate in lepton flavor violating muon to electron ($\mu^- \to e^-$) conversion process. The ANTOINE shell model code was used for calculating these properties and the results are shown in Tables 1, 2, and 3. The close agreement between obtained values and experimentally available data ensures the reliability of wave functions to use them in further calculation of NTMEs for coherent and incoherent transitions.

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ДОСЛІДЖЕННЯ СПЕКТРОСКОПІЧНИХ ВЛАСТИВОСТЕЙ ДЕЯКИХ ЯДЕР, ЩО МОЖУТЬ БРАТИ УЧАСТЬ У (μ^-, e^-) ПРОЦЕСІ ПОРУШЕННЯ ЛЕПТОННОГО ЗАРЯДУ

Порушення лептонного заряду (LFV) є явною ознакою нової фізики, що виходить за рамки Стандартної Моделі. Одним із LFV процесів є захоплення мюона з випромінюванням електрона $\mu^- \to e^-$ в мюонному атомі. У цій роботі ми досліджували спектроскопічні властивості трьох ядер, а саме ^{24}Mg, ^{32}S і ^{44}Ca, які можуть брати участь у процесі порушення лептонного заряду $\mu^- \to e^-$. Для розрахунку цих властивостей ми використали взаємодію USD для *sd* оболонкових ядер, а саме ^{24}Mg і ^{32}S, і взаємодію Z20 для *pf* оболонкового ядра ^{44}Ca.

Ключові слова: порушення лептонного заряду, перетворення мюонів в електрони, спектроскопічні властивості, джерела мюонів.

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