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AXISYMMETRIC FORCED VIBRATION OF THE HYDRO-ELASTIC SYSTEM CONSISTING OF THE PRE-STRAINED HIGHLY ELASTIC PLATE, COMPRESSIBLE INVISCID FLUID AND RIGID WALL*

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Abstract. The present paper studies the axisymmetric forced vibrations of the hydro-elastic system consisting of the plate made of a highly elastic material with axisymmetric finite initial strains, barotropic inviscid compressible fluid, and rigid wall restricting the fluid flow. The motion of the plate is described using the equations and relations of the three-dimensional linearized theory of elastic waves in bodies with initial stresses. However, the flow of the fluid is described by the linearized Euler equations for the inviscid compressible fluids. Guz's presentations for the general solution of the mentioned linearized equations are used to solve these equations for the corresponding boundary and compatibility conditions. The corresponding equations concerning these transforms are solved analytically using the Hankel integral transform. The originals of the sought values are found numerically by employing the authors' calculation algorithm and PC programs. Numerical results on the frequency response of the interface pressure are presented and discussed. In particular, it is established that the initial radial stretching of the plate leads to a decrease in the interface pressure.

Keywords: hydro-elastic plate + fluid system, highly elastic material, finite initial strains, interface pressure, compressible inviscid fluid.

Introduction.

The studies of the dynamics of plate-fluid hydro-elastic systems were started by Lamb [1], in which vibrations of a circular elastic «baffled» plate in contact with still water are considered. Since then, the related investigations have been developed, and they can be divided into the investigations related to wave dispersion problems and the investigations related to the forced vibration of the plate–fluid systems. A review of the investigations regarding wave dispersion is detailed in the papers [2 – 8]. However, a review of the investigations regarding forced vibration is detailed in the papers [9, 10, 11] and many others listed therein.

It should be noted that the investigations mentioned above can also be classified as to the accuracy of the applied plate theories and concerning the models used in describing the fluid flow. To simplify the study of the related mathematical problems, in many studies, the motion of the plate is described within the framework of various approximate plate theories, and the fluid is modeled as incompressible. Such approaches cannot describe many dynamic effects, such as the dispersion of waves, the length of which is comparable with the plate

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thickness. Moreover, these approximate theories cannot consider fluid compressibility's influence on the fluid's pressure and velocity distribution under forced vibration plate + fluid systems. However, results obtained by employing the three-dimensional linearized equations of elastodynamics for describing the plate motion for the study of the dynamics of the plate + fluid systems take into account all the above features of the studied problems, and these results can be taken as the benchmarks for the corresponding results of any approximate theories. Examples of the works above related to the wave dispersion problems and carried out within the framework of exact equations and relations can be taken from the studies made in the papers [2 – 5, 7, 8, 12 – 16], and others listed in these papers.

However, to the best of the authors' knowledge, from the historical aspect, the works related to the forced vibration and carried out within the scope of the exact equations and relations mentioned above started recently. The first considerable results in this field are obtained in the papers [17, 18] and others listed therein. These results are also discussed in the monograph [19] and are reviewed in the paper [9].

Note that in the investigations mentioned above and related to the forced vibration, the hydro-elastic system is modeled as an infinite plate + infinite compressible viscous fluid layer + rigid wall restricting the fluid flow, and it is assumed that the time-harmonic lineally located forces act on the plate. In other words, it is assumed that the plane strain state occurs in the plate, and the plane flow occurs in the fluid layer. Consequently, the results obtained within these assumptions cannot be applied to the cases where the point-located forces or the forces located in a finite region of the face plane act on the plate. According to this statement, the preceding investigations are developed in the present paper for the case where the axisymmetric point-located time-harmonic force acts on a plate made of highly elastic material. It is assumed that the pate in the initial state, i.e., before the action of the axisymmetric point-located time-harmonic force on the plate, is loaded at infinity by homogeneously distributed radial normal forces, which causes a homogeneous static axisymmetric finite strains. Thus, the present paper investigates the axisymmetric forced vibration of the hydro-elastic system consisting of the highly elastic plate with finite initial strains, compressible fluid, and rigid wall. Wherein the fluid is modeled as inviscid, and it is studied the influence of the initial strains in the plate on the fluid pressure and velocity distribution on the interface plane between the fluid and plate.

1. Formulation of the problem.

Consider a plate made of a highly elastic material with thickness h and associate the cylindrical system of coordinates $Or\theta z$ with the upper face plane of the plate (Fig. 1). We determine the position of the points of this plate with the Lagrange coordinates in this system and assume that in the initial state (before vibration of that), the plate is loaded at infinity (i.e. at $r \to \infty$) by the axisymmetric homogeneously distributed radial forces. As a result of this loading, the uniformly distributed strain-stress state appears in the plate.

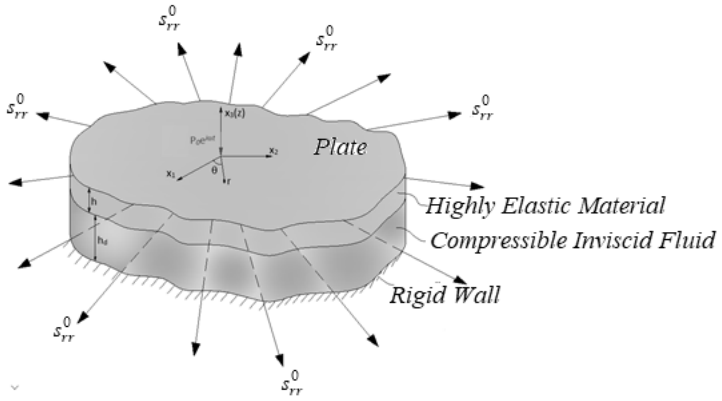


Fig. 1. The sketch of the hydro-elastic system under consideration.

According to the selected system of coordinates, before the foregoing initial loading, the plate occupies the region $\{0 \le r < \infty, 0 \le \theta \le 2\pi, -h \le z \le 0\}$, and the displacements related to the initial strain state are determined by the following expressions for the components of the displacement vector:

$$u_r^0 = (\lambda_1 - 1)r; \quad u_\theta^0 = 0; \quad u_z^0 = (\lambda_3 - 1)z, \tag{1}$$

where λ_1 and λ_3 are constants and are called elongation parameters. In (1) and below, the upper index 0 denotes that the related quantity belongs to the initial strain-stress state.

Assume that the material of the plate is described by the harmonic potential proposed by John [20] which has the following expression:

$$\Phi = \frac{1}{2}\lambda e_1^2 + \mu e_2, \tag{2}$$

where

$$e_1 = \sqrt{1+2\varepsilon_1} + \sqrt{1+2\varepsilon_2} + \sqrt{1+2\varepsilon_3} - 3;$$

$$e_2 = \left(\sqrt{1+2\varepsilon_1} - 1\right)^2 + \left(\sqrt{1+2\varepsilon_2} - 1\right)^2 + \left(\sqrt{1+2\varepsilon_3} - 1\right)^2. \tag{3}$$

In relations (2) and (3), λ and μ are material constants and ε_i $(i = 1,2,3)$ are the principal values of Green's strain tensor.

The physical components of the symmetric stress tensor $s_{(ij)}$ in the non-linear theory of elasticity for highly elastic (or hyper-elastic) materials are determined by the following expression:

$$s_{(ij)} = \frac{1}{2}\left(\frac{\partial}{\partial \varepsilon_{(ij)}} + \frac{\partial}{\partial \varepsilon_{(ji)}}\right)\Phi, \tag{4}$$

where under (ij) the indices $rr, \theta\theta, zz, r\theta, rz, z\theta$ are understood.

Doing the procedures described in [4, 7, 9], the following expressions are obtained for the components of the stress tensor in the initial state described by (1):

$$s_{rr}^0 = s_{\theta\theta}^0 = \frac{2\mu}{\lambda_1}(\lambda_1 - \lambda_3); \quad s_{zz}^0 = s_{r\theta}^0 = s_{z\theta}^0 = s_{rz}^0 = 0, \tag{5}$$

where

$$\lambda_3 = \frac{\lambda(3 - 2\lambda_1) + 2\mu}{\lambda + 2\mu} = \frac{(3 - 2\lambda_1) + 2\mu/\lambda}{1 + 2\mu/\lambda}. \tag{6}$$

It follows from (5) and (6) that, for the selected material constants λ and μ, the initial state is determined only by the elongation parameter λ_1, i.e., the magnitude of the initial strain-stress state can be estimated only through the parameter λ_1.

Assume that after appearing the foregoing initial strain-stress state in the plate that is in contact with the fluid, the axisymmetric forced vibration takes place in the hydro-elastic system. For writing the equations and relations of this vibration, we use the Lagrange coordinates r' and z' in the coordinate system $O'r'\theta'z'$ related to the foregoing initial state, and these coordinates are determined through the coordinates r and z by the following expressions:

$$r' = \lambda_1 r; \quad z' = \lambda_3 z, \tag{7}$$

according to which, the thickness of the plate becomes h' where $h' = \lambda_3 h$.

Within the framework of the foregoing assumptions and notation, and according to the monograph [21], we write the equations and relations of the three-dimensional linearized theory of elastic waves in initially finite pre-strained elastic bodies with the Lagrange coordinates r' and z'.

The linearized equations of motion:

$$\frac{\partial Q_{r'r'}}{\partial r'}+\frac{\partial Q_{z'r'}}{\partial z'}+\frac{1}{r'}\left(Q_{r'r'}-Q_{\theta'\theta'}\right)=\rho'\frac{\partial^2 u_{r'}}{\partial t^2};$$
$$\frac{\partial Q_{r'z'}}{\partial r'}+\frac{\partial Q_{z'z'}}{\partial z'}+\frac{1}{r'}Q_{r'z'}=\rho'\frac{\partial^2 u_{z'}}{\partial t^2}. \tag{8}$$

The linearized elasticity relations:

$$Q_{r'r'}=\omega'_{1111}\frac{\partial u_{r'}}{\partial r'}+\omega'_{1122}\frac{u_{r'}}{r'}+\omega'_{1133}\frac{\partial u_{z'}}{\partial z'};$$
$$Q_{z'z'}=\omega'_{3311}\frac{\partial u_{r'}}{\partial r'}+\omega'_{3322}\frac{u_{r'}}{r'}+\omega'_{3333}\frac{\partial u_{z'}}{\partial z'};\ Q_{r'z'}=\omega'_{1313}\frac{\partial u_{r'}}{\partial z'}+\omega'_{1331}\frac{\partial u_{z'}}{\partial r'}; \tag{9}$$
$$Q_{z'r'}=\omega'_{3113}\frac{\partial u_{r'}}{\partial z'}+\omega'_{3131}\frac{\partial u_{z'}}{\partial r'}.$$

In (8) and (9), $Q_{r'r'}$, $Q_{\theta'\theta'}$, …, $Q_{z'r'}$ are the perturbations of the components of the non-symmetric Kirchhoff stress tensor, $u_{r'}$ and $u_{z'}$ are the perturbations of the components of the displacement vector in the system of coordinates $O'r'\theta'z'$, ρ' is the plate material density related to the unit volume in the initially strained state and $\rho'=(\lambda_1^2\lambda_3)^{-1}\rho$ where ρ is the plate material density in its natural state. Moreover, according to [21], in the case under consideration, the components $\omega'_{1111},\omega'_{1122},\ldots,\omega'_{3131}$ are determined by the following expressions:

$$\omega'_{1111}=(\lambda+2\mu)\frac{1}{\lambda_3};\ \omega'_{2222}=\omega'_{1111};\ \omega'_{3333}=\frac{\lambda_3}{\lambda_1^2}(\lambda+2\mu);\ \omega'_{1122}=\frac{1}{\lambda_3}\lambda;$$
$$\omega'_{1133}=\frac{1}{\lambda_1}\lambda;\ \omega'_{1313}=2\mu\frac{\lambda_3}{\lambda_1(\lambda_1+\lambda_3)};\ \omega'_{3113}=\omega'_{1331}=2\mu\frac{\lambda_1}{\lambda_3(\lambda_1+\lambda_3)}. \tag{10}$$

This completes the field equations and relations within the framework of which the vibration of the plate is described.

Now we consider the linearized Euler equations describing the flow of the barotropic compressible viscous fluid. These equations in terms of the velocities are:

$$\rho_{01}\frac{\partial}{\partial t}V_{r'}+\frac{\partial}{\partial r'}p_1=0;\quad \rho_{01}\frac{\partial}{\partial t}V_{z'}+\frac{\partial}{\partial z'}p_1=0,, \tag{11}$$

where ρ_{01} is the fluid density before the perturbation, p_1 is the perturbation of the pressure, and $V_{r'}$ and $V_{z'}$ are the components of the perturbation velocity vector.

It must be added to (11) the following linearized continuity and state equations, and corresponding rheological relations.

The linearized continuity equation:

$$\frac{\partial\rho_1}{\partial t}+\rho_{01}\left(\frac{\partial V_r}{\partial r}+\frac{V_r}{r}+\frac{\partial V_z}{\partial z}\right)=0. \tag{12}$$

The linearized state equation:

$$p_1=a_0^2\rho_1;\quad a_0^2=\left(\frac{\partial p_1}{\partial\rho_1}\right)_0, \tag{13}$$

where a_0 is the sound speed in the fluid, and ρ_1 is the perturbation of the fluid density.

Note that equations (11) (13) are written according to the monograph [22] in the Euler coordinates r' and z' in the $Or'\theta'z'$ coordinate system. At the same time, taking the small-

ness of the perturbations into account, the difference between the Euler and Lagrange coordinates is ignored.

Consider formulation of the compatibility and boundary conditions for the hydro-elastic system under consideration and begin this writing with the following boundary conditions on the upper face plane (i.e. at $z'=0$) of the plate:

$$Q_{z'z'}\big|_{z'=0} = -P\delta(r')e^{i\omega t}; \quad Q_{r'z'}\big|_{z'=0} = 0. \tag{14}$$

The mentioned compatibility conditions are:

$$Q_{z'z'}\big|_{z'=-h'} = -\,p_1\big|_{z'=-h'}; \quad Q_{r'z'}\big|_{z'=-h'} = 0; \quad \left.\frac{\partial u_{z'}}{\partial t}\right|_{z'=-h'} = V_{z'}\big|_{z'=-h'}. \tag{15}$$

The impermeability conditions on the rigid wall are:

$$V_{z'}\big|_{z'=-h'-h_d} = 0. \tag{16}$$

Besides all of these conditions, it is required satisfaction of the following decay conditions:

$$\left\{|Q_{rr}|; |Q_{\theta\theta}|; |Q_{zz}|; |Q_{rz}|; p_1; |u_r|; |u_z|; |V_r|; |V_z|\right\} \to 0 \text{ as } r \to \infty. \tag{17}$$

This completes the formulation of the problem.

2. Method of solution.

For solution to the system of equations (8) and (9) we use the presentations:

$$u_{r'} = -\frac{\partial^2}{\partial r'\partial z'}\mathrm{X}; \quad u_{z'} = \frac{1}{\omega'_{1111}+\omega'_{1313}}\left(\omega'_{1111}\Delta'_1 + \omega'_{3113}\frac{\partial^2}{\partial z'^2} - \rho'\frac{\partial^2}{\partial t^2}\right)\mathrm{X}; \tag{18}$$

proposed in [21], where the function X is determined from the equation

$$\left[\left(\Delta' + (\xi'_2)^2\frac{\partial^2}{\partial z'^2}\right)\left(\Delta' + (\xi'_3)^2\frac{\partial^2}{\partial z'^2}\right) - \rho'\left(\frac{\omega'_{1111}+\omega'_{1331}}{\omega'_{1111}\omega'_{1331}}\Delta' + \frac{\omega'_{3333}+\omega'_{3113}}{\omega'_{1111}\omega'_{1331}}\right)\frac{\partial^2}{\partial t^2} + \right.$$

$$\left. + \frac{\rho'}{+\omega'_{1111}\omega'_{1331}}\frac{\partial^4}{\partial t^4}\right]\mathrm{X} = 0; \quad \Delta' = \frac{d^2}{dr'^2} + \frac{1}{r'}\frac{d}{dr'}, \tag{19}$$

where

$$(\xi'_{2,3})^2 = d \pm \left[d^2 - \omega'_{3333}\omega'_{3113}\left(\omega'_{1111}\omega'_{1331}\right)^{-1}\right]^{1/2};$$

$$d = \left[\omega'_{1111}\omega'_{3333} + \omega'_{1331}\omega'_{3113} - (\omega'_{1133}+\omega'_{1313})^2\right]\left(2\omega'_{1111}\omega'_{1331}\right)^{-1}. \tag{20}$$

Before considering the solution to equation (19), according to the boundary conditions in (14) and the nature of the problem under consideration, we represent all sought quantities as $g(r',z',t) = \overline{g}(r',z')e^{i\omega t}$, where $\overline{g}(r',z')$ is the amplitude of the corresponding quantity. Below we omit the overbar on the amplitude symbols and use the symbol for the amplitude, which is the same as the symbol of the corresponding quantity.

In this way, we obtain the following equation for the X amplitude from equation (19) using the above-noted presentation for the sought values:

$$\left[\left(\Delta_1 + \xi_2^2\frac{\partial^2}{\partial z^2}\right)\left(\Delta_1 + \xi_3^2\frac{\partial^2}{\partial z^2}\right) + \frac{\omega^2h^2}{c_2^2}\left(\frac{\lambda/\mu+3}{\lambda/\mu+2}\right)\left(\Delta_1 + \frac{\partial^2}{\partial z^2}\right) + \frac{\omega^4h^4}{c_2^4(\lambda/\mu+2)}\right]\mathrm{X} = 0, \tag{21}$$

where $c_2 = \sqrt{\mu/\rho}$.

Note that in writing the equation (21), we have used the dimensionless coordinates $\overline{r} = r/h$ and $\overline{z} = z/h$ after the overbars of the dimensionless coordinates have been omitted. Below we also use these dimensionless coordinates.

Thus, we consider the solution procedure to equation (19) for which we use the Hankel integral presentation for the function X:

$$\mathrm{X} = \int_0^\infty F(k) e^{\chi z'} J_0(kr')kdk, \tag{22}$$

where $J_0(kr')$ is the first kind of Bessel function with the zeroth order.

Substituting (22) into (19), with some mathematical manipulation, we conclude that in order to satisfy equation (19), χ must satisfy the algebraic equation:

$$A_1\chi^4 + B_1\chi^2 + C_1 = 0, \tag{23}$$

where

$$A_1 = \xi_2^2\xi_3^2; \quad B_1 = \frac{\lambda/\mu+3}{\lambda/\mu+2}\frac{(h\omega)^2}{c_2^2} - k^2(\xi_2^2+\xi_3^2);$$

$$C_1 = -k^2\frac{\lambda/\mu+3}{\lambda/\mu+2}\frac{(h\omega)^2}{c_2^2} + \frac{(h\omega)^4}{c_2^4(\lambda/\mu+2)} + k^4. \tag{24}$$

Consequently, the roots of equation (23) we determine as follows:

$$\chi_1 = \sqrt{-\frac{B_1}{2A_1} + \sqrt{\frac{B_1^2}{4A_1^2} - \frac{C_1}{A_1}}}; \quad \chi_2 = -\sqrt{-\frac{B_1}{2A_1} + \sqrt{\frac{B_1^2}{4A_1^2} - \frac{C_1}{A_1}}}; \quad \chi_3 = -\chi_1; \quad \chi_4 = -\chi_2. \tag{25}$$

Thus, we determine the following expression for the function X:

$$\mathrm{X} = \int_0^\infty \left[F_1 e^{\chi_1 z'} + F_2 e^{-\chi_1 z'} + F_3 e^{\chi_2 z'} + F_4 e^{-\chi_2 z'}\right] J_0(kr')kdk. \tag{26}$$

For simplicity of use of the dimensionless coordinates, we change the unknown constants F_n with $h'^3\overline{F}_n$, where $(n = 1,2,3,4)$ and below we omit the overbar in $\overline{F}_n$. Thus, we determine the following formulae for the displacements by substituting expression (26) into the presentations in (18):

$$u_{r'} = h'\int_0^\infty \left[F_1\chi_1 e^{\chi_1 z'} - F_2\chi_1 e^{-\chi_1 z'} + F_3\chi_2 e^{\chi_2 z'} - F_4\chi_2 e^{-\chi_2 z'}\right] k^2 J_1(kr')dk;$$

$$u_{z'} = h'\int_0^\infty \left\{\left[-k^2\frac{\lambda/\mu+2}{\lambda/\mu+1} + \frac{1}{\lambda/\mu+1}\left(\chi_1^2 + \frac{(\omega h)^2}{c_2^2}\right)\right]\left(F_1 e^{\chi_1 z'} + F_2 e^{-\chi_1 z'}\right) + \right.$$

$$\left. + \left[-k^2\frac{\lambda/\mu+2}{\lambda/\mu+1} + \frac{1}{\lambda/\mu+1}\left(\chi_2^2 + \frac{(\omega h)^2}{c_2^2}\right)\right]\left(F_3 e^{\chi_2 z'} + F_4 e^{-\chi_2 z'}\right)\right\} J_0(kr')kdk. \tag{27}$$

Using the elasticity relations (9) and expressions in (27), we determine the expressions for the stresses which enter the boundary and compatibility conditions. To reduce the volume of the paper we do not present here these expressions the obtaining of which requires obvious mathematical calculations.

In this way, we determine the expressions of the Hankel integral presentations for the displacements and stresses related to the plate. These expressions contain the unknown constants F_1, F_2, F_3, and F_4, to the determination of which we turn below.

To solve the field equations related to the fluid flow, according to the monograph [22], we use presentations for the general solution to the linearized Euler equations:

$$V_{r'} = \frac{\partial}{\partial r'}\Phi; \quad V_{z'} = \frac{\partial}{\partial z'}\Phi; \quad p_1 = -\rho_{01}\frac{\partial}{\partial t}\Phi; \quad \rho_1 = -\frac{\rho_{01}}{a_0^2}\frac{\partial}{\partial t}\Phi, \tag{28}$$

where the functions Φ and Ψ satisfy the following equations:

$$\left[\Delta - \frac{1}{a_0^2}\frac{\partial^2}{\partial t^2}\right]\Phi = 0; \quad \Delta = \frac{\partial^2}{\partial r'^2} + \frac{1}{r'}\frac{\partial}{\partial r'} + \frac{\partial^2}{\partial z'^2}. \tag{29}$$

As above, we represent the functions Φ as $\Phi(r', z', t) = \overline{\Phi}(r', z')e^{i\omega t}$, where $\overline{\Phi}(r', z')$ (below we omit the overbar on $\overline{\Phi}(r', z')$) is the amplitude of the Φ. For simplicity of use of the dimensionless coordinates we introduce the following presentation $\Phi = \omega h^2\varphi$ for the function Φ. Thus, we use the Hankel integral presentation $\varphi = \int_0^\infty Ge^{\delta z'}J_0(kr')kdk$ and, substituting this presentation into the equation (29) and doing some mathematical calculations, we determine the following expression for the Hankel integral presentation of the function Φ:

$$\Phi = \omega h^2\int_0^\infty\left(G_1e^{\delta_1 z'} + G_2e^{-\delta_1 z'}\right)J_0(kr')kdk, \tag{30}$$

where

$$\delta_{1,2} = \pm\sqrt{k^2 - \Omega_1^2}; \quad \Omega_1 = \frac{\omega h}{a_0}. \tag{31}$$

Thus, substituting the expressions (30) into the expressions in (28), we obtain the following Hankel integral presentations for the velocities:

$$V_r = -\omega h\int_0^\infty\left[G_1e^{\delta_1 z} + G_2e^{-\delta_1 z}\right]k^2J_1(kr)dk; \quad V_z = \omega h\int_0^\infty\left[G_1\delta_1e^{\delta_1 z} - G_2\delta_1e^{-\delta_1 z}\right]kJ_0(kr)dk, \tag{32}$$

where G_1, and G_2 are unknown constants.

Now we consider the determination of the unknow constants F_1, F_2, F_3, and F_4 which enter into the expressions of the stresses and displacements of the plate, and the unknown constants G_1, and G_2. Under obtaining the algebraic equations containing these unknowns, it is necessary to obtain the Hankel transform of the function $P\delta(r')$ which is on the left side in the first boundary condition in (14). For this purpose, we present the function $P\delta(r')$ as $\lim_{r'\to 0} P_0/(h'^2\pi r'^2)$ (here r' is a dimensionless coordinate and P_0 has the force dimension, however, P has the stress dimension, i.e. P is presented as P_0/h'^2). From $\lim_{\varepsilon\to 0}\int_0^\varepsilon \frac{P_0J_0(kr')r'dr'}{h'^2\pi\varepsilon^2}$ we obtain $\frac{P_0}{h'^2 2\pi}$ for the Hankel transform of $P\delta(r')$. After this preparation, we obtain the following equations concerning the above unknown constants:

$$Q_{z'z'}\big|_{z'=0} = -P\delta(r') \Rightarrow \sum_{n=1}^4 a_{1n}F_n = -\frac{P_0}{h'^2 2\pi}; \quad Q_{z'r'}\big|_{z'=0} = 0 \Rightarrow \sum_{n=1}^4 a_{2n}F_n = 0;$$

$$Q_{z'z'}\big|_{z'=-h'} + p_1\big|_{z'=-h'} = 0 \Rightarrow \sum_{n=1}^4 a_{3n}F_n - \sum_{n=1}^4 b_{3n}G_n = 0;$$

$$Q_{z'r'}\big|_{z'=-h'}=0\Rightarrow\sum_{n=1}^{4}a_{4n}F_n=0;\quad\left.\frac{\partial u_{z'}}{\partial t}\right|_{z'=-h'}-V_{z'}\big|_{z'=-h'}=0\Rightarrow\sum_{n=1}^{4}a_{5n}F_n-\sum_{n=1}^{4}b_{5n}G_n=0;$$

$$V_{z'}\big|_{z'=-h'-h_d}=0\Rightarrow\sum_{n=1}^{4}b_{6n}G_n=0. \tag{33}$$

Note that the explicit expressions of the coefficients a_{jn} and b_{mn} $(n=1,2,3,4)$, $(j=1,2,\dots,6)$, $(m=3,4,\dots,8)$ are not given here and they can easily be determined from the expressions of the foregoing velocities and stresses.

After determining the unknown constants above, we determine the Hankel integral presentations of all the sought quantities. The determination of the originals of the sought values from these presentations is made numerically by the algorithm, which we will discuss in the next section.

3. Numerical results and discussions.

Under calculation procedures, the improper integrals $\int_0^{\infty}(\bullet)dk$ such as in (27), and (32) are replaced by the corresponding definite integrals $\int_0^{S_1^*}(\bullet)dk$. The values of S_1^* are determined from the convergence requirement of the numerical results. Note that under calculation of the latter integrals, the integration intervals are further divided into a certain N number of shorter intervals, which are used in the Gauss integration algorithm. The values of the integrated expressions at the sample points are calculated through the equations in (33). All these procedures are performed automatically with the PC programs constructed by the authors in MATLAB.

Note that detailed discussions of the calculation algorithm and its convergence are made in the papers [17, 18] and others listed therein. Therefore, we do not consider these questions here and note that all the numerical results discussed below are obtained in the case where $N=10000$ and $S_1^*=5$.

Thus, consider some numerical results for the frequency response of the interface pressure obtained within the foregoing solution method in the case where the plate material is soft-rubber with mechanical constants $\mu=1,2*10^6\,\text{Pa}$; $\lambda=58,8*10^6\,\text{Pa}$; $\rho=1200\,\text{kg/m}^3$ and $c_2=\sqrt{\mu/\rho}=31,622\,\text{m/s}$ [21], and the material of the fluid is Glycerin with a density $\rho=1260\,\text{kg/m}^3$ and sound speed $a_0=1927\,\text{m/sec}$ [22].

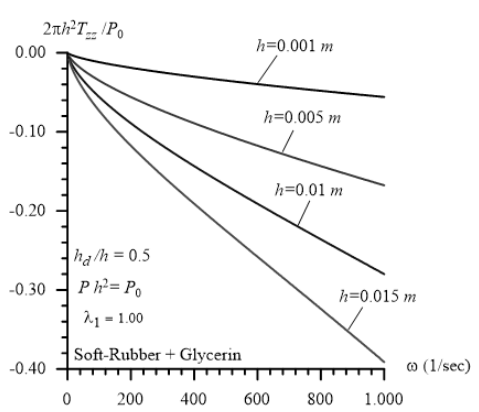


Fig. 2

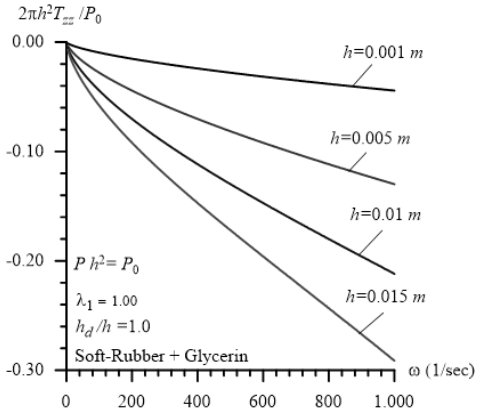


Fig. 3

The graphs of the mentioned frequency responses obtained in the case where the initial strains in the plate are absent are presented in Figs. 2 and 3 for various values of the plate thickness h under $h_d/h = 0,5$ (Fig. 2) and 1,0 (Fig. 3). Note that in these figures, $T_{zz} = -p_1$, where p_1 is the interface pressure and under construction, it is assumed that $\omega t = 0$. Such selection of the ωt is because the pressure has its absolute maximum values in the cases where the vibration phase has values determined by the expression $\omega t = 0 + n\pi$ ($n = 0; \pm 1; \pm 2; ...$), which is valid for the inviscid fluid case. Thus, these results show that an increase in the values of the plate thickness h causes an increase in the absolute values of the interface pressure. Moreover, these results and other ones not given here show that an increase in the fluid layer depth, i.e., in the values of the ratio h_d/h, leads to a decrease in the absolute values of the pressure. In the qualitative sense, these results agree with the corresponding results obtained in the papers [17, 18] and the well-known physical-mechanical considerations.

We recall that one of the primary purposes of the present numerical investigations is to determine how the plate's initial stretching affects the character of the frequency responses under consideration in the qualitative and quantitative sense. Also, we recall that the magnitude of the axisymmetric initial radial stretching of the plate is estimated through the parameter λ_1. According to expression (6), an increase in the values of the parameter λ_1 causes a decrease in the values of the parameter λ_3. Consequently, a decrease in the values of the λ_3 leads to a decrease in the thickness $h'(= \lambda_3 h)$ of the plate in the initial state. In order to $h' > 0$, it must be $\lambda_1 < 1,5$, for which $\lambda_3 > 0$. Even in the cases where $\lambda_1 < 1,5$ for the values of λ_1, which are approximately equal to 1,25, 1,4, and 1,45, the values of the h' becomes equal to $0,5h$, $0,2h$, and $0,1h$, respectively, under which the plate in the initial state becomes very thin for the selected above values of h (see, Figs. 2 and 3). It may be assumed that such cases cannot have real physical meaning in the application sense, and therefore, below, we consider only the results obtained for the cases where $\lambda_1 \le 1,2$.

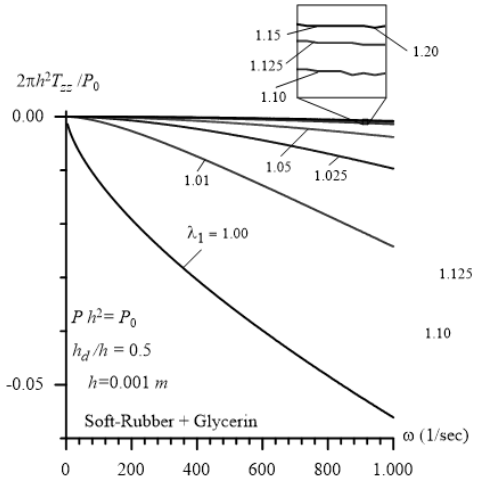


Fig. 4

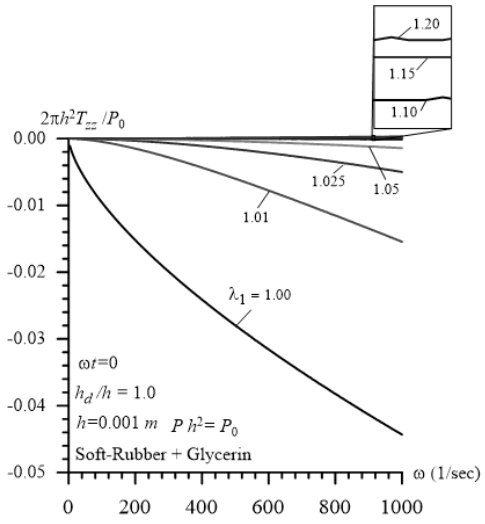


Fig. 5

Thus, we analyze numerical results related to the influence of the initial stretching of the plate, i.e., of the parameter λ_1, on the frequency response of the interface pressure. Note that the related results obtained under $h_d/h = 0,5$, 1,0, and 2,0 for the case where $h = 0,001\,\text{m}$ (for the case where $h = 0,015\,m$) are given in Figs. 4, 5, and 6 (in Figs. 7, 8 and 9) respectively.

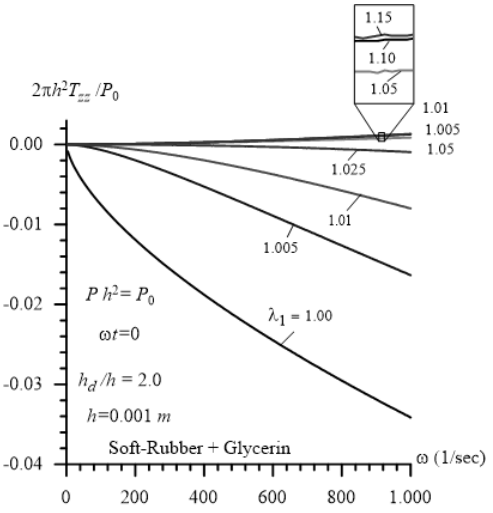


Fig. 6

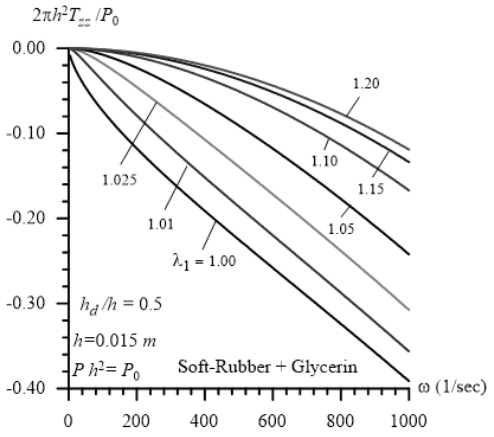


Fig. 7

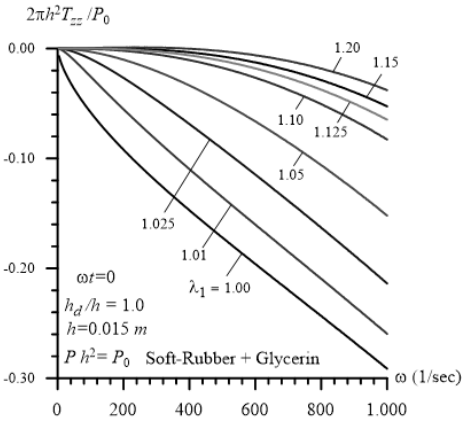


Fig. 8

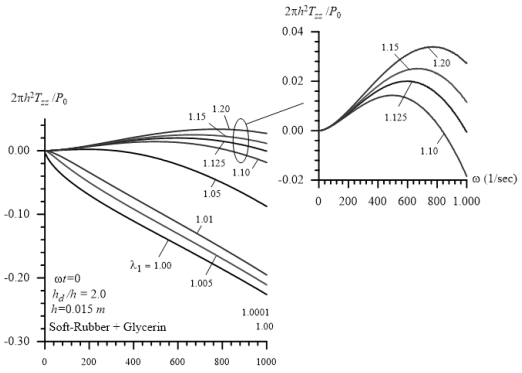


Fig. 9

Analysis of the results given in Figs. 4, 5, and 6 show that the initial stretching of the plate leads to a decrease in the absolute values of the interface pressure. Moreover, comparing the results given in each figure with those given in other figures shows that an increase in the values of the h_d/h, i.e., an increase of the fluid depth, causes a decrease in the values of this pressure. The dependence between the pressure values and parameter λ_1 has a monotonic character for the cases under consideration, i.e., for the cases where $1 \le \lambda_1 \le 1,20$. However, the magnitude of the increase in pressure values with the parameter λ_1 decreases with the this parameter. In the near vicinity of the value $\lambda_1 = 1,20$, the mentioned increase almost stops for the case where $h = 0,001m$. The numerical results not given here show that after a certain value of λ_1 (denote it by λ_1^*), an increase in the values of λ_1 causes a decrease in the values of the pressure. In other words, in the cases where $\lambda_1^* < \lambda_1 < (3+2\mu/\lambda)/2$, an increase in the values of the λ_1 causes a decrease in the pressure under consideration. According to the discussions above, the results obtained in the cases where $\lambda_1^* < \lambda_1 < (3+2\mu/\lambda)/2$ are not considered here.

Now we analyze the results related to the thicker plate, i.e., to the case where $h = 0,015\,\text{m}$ and given in Figs. 7, 8, and 9. These results show that an increase in the plate thickness decreases the intensity of the pressure decrease with λ_1 in the case $h = 0,015\,\text{m}$, which is less than that in the case $h = 0,001\,\text{m}$. According to the no complicated mechanical

considerations, these results can also predicted before obtaining those. In other words, these results agree well with the well-known mechanical considerations.

Moreover, these results show that in the case $h = 0,015\,\text{m}$, the character of the frequency response of the pressure depends significantly on the λ_1 and h_d/h. What is more, the change in the character of the mentioned dependencies takes place not only in the quantitative but also in the qualitative sense. Also, the analyses of the results given in Figs. 7, 8, and 9, show that the influence of the parameter λ_1 on the frequency responses has the same character, which is in the case where $h = 0,001\,\text{m}$. However, in the present case, the value of the λ_1^* is more than that obtained in the case $h = 0,001\,\text{m}$.

It should be noted that in the preceding discussions under the "pressure," it is understood that the perturbation of the pressure in the fluid, the magnitude of which is significantly less than the initial pressure in the fluid. Therefore, the results obtained in the relatively great values of the parameter λ_1, i.e., under $1,10 \le \lambda_1 \le 1,20$ in the case where $h_d / h = 2.0$ the values of the pressure perturbation become no positive (Fig. 9) and dependence between the pressure and frequency becomes no monotonic unlike to the other considered cases under which this dependence has monotonic character.

With this we restrict ourselves to analyze the numerical results related to the frequency response of the interface pressure.

Conclusions.

Thus, the present paper studies the axisymmetric forced vibration of the hydro-elastic system consisting of the plate made of a highly elastic material with axisymmetric finite initial strains, barotropic inviscid compressible fluid, and rigid wall restricting the fluid flow. Under this study, the motion of the plate is described with the equations and relations of the three-dimensional linearized theory of elastic waves in bodies with initial stresses. However, the flow of the fluid is described by the linearized Euler equations for the inviscid compressible fluids. Guz's presentations for the general solution of the mentioned linearized equations are used to solve these equations for the corresponding boundary and compatibility conditions. The corresponding equations concerning these transforms are solved analytically using the Hankel integral transform. The originals of the sought values are found numerically by employing the authors' calculation algorithm and PC programs.

Numerical results on the frequency response of the interface pressure are presented for the use of the harmonic potential for describing the plate material, but as a fluid glycerin is selected. According to these results, it can be drawn the following concrete results:

– an increase in the thickness of the plate leads to an increase in the interface pressure;

– the initial radial stretching of the plate causes a decrease in the magnitude of the interface pressure;

– the intensity of the mentioned increase of the interface pressure decreases with the initial stretching of the plate, and this increase stops under a certain value of the initial stretching;

– the character of the dependence between the interface pressure and frequency depends significantly on the plate thickness and the fluid depths. This dependence has not only quantitative but also qualitative character.

Finally, we note that the investigations made in the present paper are the beginning of the corresponding investigations in the related field of the dynamics of the hydro-elastic system. The authors will further develop the corresponding investigations on the case where the fluid is modeled as a compressible viscous.

РЕЗЮМЕ. У даній роботі досліджено осесиметричні вимушені коливання гідропружної системи, що складається з пластини з високоеластичного матеріалу з осесиметричними скінченними початковими деформаціями, баротропної нев'язкої стисливої рідини та жорсткої стінки, що обмежує потік рідини. Рух пластини описується за допомогою рівнянь і співвідношень тривимірної лінеаризованої теорії пружних хвиль у тілах з початковими напруженнями. Однак течія рідини описується лінеаризованими рівняннями Ейлера для нев'язких стисливих рідин. Представлення Гузя загального розв'язку згаданих лінеаризованих рівнянь використовуються для розв'язування цих рівнянь для відповідних граничних умов та умов сумісності. Відповідні рівняння щодо цих перетворень розв'язуються аналітично за допомогою інтегрального перетворення Ханкеля. Оригінали знаходять чисельно за допомогою авторського алгоритму розрахунку та програм для ПК. Наведено та обговорено числові результати частотної характеристики тиску на межі поділу. Зокрема, встановлено, що початковий радіальний розтяг пластини призводить до зменшення тиску на межі поділу.

КЛЮЧОВІ СЛОВА: гідропружна система «пластина + рідина», високоеластичний матеріал, скінченні початкові деформації, тиск на поверхні поділу, стислива нев'язка рідина.

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