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DECISION MAKING IN MINING INDUSTRY BASED ON THEORY OF GAMES WITH NATURE UNDER INTERVAL DATA

Abstract. The need for decision-making is present in all areas of human activity (sociology, medicine, economics, military affairs, etc.). This is also extremely important in mining industry. Theory of games is widely used to support decision-making. Theory of games is an effective decision-making tool in the planning and management of mining enterprises. The problem of decision-making under uncertainty, defined as the absence or insufficiency of information, is considered. In this case, it is appropriate to use the mathematical model "Playing with Nature." In this model, the conscious "player" is the decision maker, and "nature" is the environment surrounding the system, and there is no information about the probabilities of nature states. Most games are analyzed with deterministic data (payoff matrices). However, it is practically impossible to determine them precisely. This is due to many factors. One of them is that the data is not fully determined by the strategies of the game participants, but also depends on many other factors. A distinctive feature of games with interval data is that the payoffs are not precisely defined. For these games, only the upper and lower bounds of possible values are known, or, equivalently, only the intervals within which the payoff values may lie. A matrix in which payoffs are assigned intervals of possible values rather than specific numbers more closely corresponds to reality. Studies examining games with interval data are primarily devoted to identifying the existence of a saddle point and developing methods for finding it. The following classical criteria are most often used to select the optimal strategy: Wald maximin and maximax. However, they do not take into account the interval distributions of all payoffs under various states of nature. The purpose of this work is to improve the application of the Wald and Maximax criteria by taking into account the influence of all states of nature on the choice of a decision-making strategy for interval data. Applications of the Wald and maximax criteria, calculated using the lower and upper values of payoff intervals, respectively, are analyzed. If a decision maker accepts a reduction in the criterion within acceptable limits, then it is possible to select a strategy superior to the strategy determined based on the criterion. When using the Wald criterion, the increments of payoffs for each strategy are additionally determined compared to the minimum possible, and are linearly averaged. When comparing strategies, the one with the larger mean deviation is considered more profitable. When using the maximax criterion, the decreases in payoffs compared to the maximum possible are additionally determined and linearly averaged. When comparing strategies, the one with the smaller mean deviation is considered more profitable.

Keywords: game, nature, uncertainty, interval, criteria, mean deviation.

1. Introduction

The need for decision-making is present in all areas of human activity (sociology, medicine, economics, military affairs, etc.). This is also extremely important in mining industry. Theory of games is widely used to support decision-making [1–7]. Theory of games is an effective decision-making tool in the planning and management of mining enterprises [4–6].

A review [7] notes that the number of books and articles on theory of games amounts to many thousands. The history of the development of theory of games is given in [8].

Let us consider the problem of decision making under uncertainty [9–12]. Uncertainty is defined as the absence or insufficiency of information [10]. In this case, it is appropriate to use the mathematical model "Playing with Nature." In this model, the conscious "player" is the decision maker, and "nature" is the environment surrounding the system, and there is no information about the probabilities of nature



states. Most games analyzed involve deterministic data (payoff matrices). However, it is practically impossible to determine these data precisely. This is due to many factors. One of them is that the data are not fully determined by the strategies of the game participants, but also depend on many other factors.

A distinctive feature of interval games is that the payoffs are not precisely defined. Only the upper and lower bounds of possible values are known, or, equivalently, only the intervals within which the payoff values may lie. A matrix in which payoffs are assigned intervals of possible values rather than specific numbers more accurately reflects reality.

Studies examining games with interval data (reviews are provided in [2, 7, 13]) are primarily devoted to identifying the existence of a saddle point and developing methods for finding it. An analysis of cooperative games is presented in [14].

When using a game with nature, it has been proposed [15] to replace the interval with the mean value of a certain preference function. However, the choice of this function is at the discretion of the decision maker, i.e., the uncertainty of the initial data is transformed into uncertainty of the form. Furthermore, such a replacement is sensitive to the interval width. The following classical criteria are most often used to select the optimal strategy: Wald maximin and maximax . [9, 17]. However, they do not take into account the interval distributions of all payoffs under various states of nature.

Therefore, the purpose of this study is improvement the application of the Wald and maximax criteria by taking into account the influence of all states of nature on the choice of decision-making strategy for interval data.

Object of the study: the conditions for applying the Wald and maximax criteria, taking into account the influence of all states of nature on the choice of decision-making strategy for interval data.

2. Methods

The following methods were used in this study: an analytical review of literature sources; methods of theory of games; and fuzzy set methods. Internet resources were used to analyze the current state of the art.

To achieve the set purpose, the following tasks were solved:

- to consider the application of the Wald and maximax criteria, which were calculated, respectively, based on the lower and upper values of the payoff intervals;
- when using the Wald criterion for each strategy, additionally determine the increase in payoffs compared to the minimum possible and perform their linear averaging;
- when applying the maximax criterion, additionally determine the decrease in payoffs compared to the maximum possible and perform their linear averaging.

3. Theoretical and experimental parts

Let us assume that the decision maker has m strategies, and nature has n states. A decision maker knows the payoffs values a_{ij} at i strategies S_i ($i = \overline{1, m}$) and

j state of nature ($j = \overline{1, n}$). A game with interval data differs from a game with deterministic data in that the elements of the payoff matrix are the closed intervals $a_{ij} = [\underline{a}_{ij}, \bar{a}_{ij}]$, where $\underline{a}_{ij}$ and $\bar{a}_{ij}$ are the lower and upper limits of the interval [19]. A totality of elements a_{ij} form the matrix of the game A of size $m \times n$.

We'll consider a situation where the decision maker has only one attempt to make a decision. In other words, relying on interval averaging is unacceptable.

We will determine the optimal strategy using the Wald criterion (the criterion of guaranteed result, the pessimism criterion). This is one of the most important criteria for reliable decision-making [5, 11–13]. It is also used in a number of composite criteria: Hurwitz, Hodge-Lehmann, and Germeier-Hurwitz [9, 16, 17].

Scope of application of the Wald criterion [9, 18]:

- nothing is known about the probabilities of states of nature;
- we will not allow risk;
- the decision is made once.

The Wald criterion is based on a comparison of minimum payoffs, therefore, to ensure the applicability of the criterion regardless of the relative position of the intervals, taking into account A , we form a matrix of lower boundary values for each strategy S_i and perform a permutation (ranking) of the payoffs $\underline{a}_{ij}$ by arranging them in non-decreasing order [6]. We denote the elements of the resulting matrix as $\underline{b}_{ij}$, and the matrix itself as $\underline{B}$.

Thus, in each i -th row of the matrix $\underline{B}$ we have:

$$\underline{b}_{i1} \leq \underline{b}_{i2} \leq \dots \leq \underline{b}_{in}. \quad (1)$$

By virtue of relation (1), the first column consists of minimum payoffs

$$\underline{b}_{i1} = \min_j \underline{a}_{ij}.$$

Therefore, the Wald criterion is

$$W = \max_i \underline{b}_{i1} = \max_i \min_j \underline{a}_{ij}.$$

We denote the strategy, for which the Wald criterion is valid, as S_w .

In addition to the traditional approach, we introduce into consideration the permissible interval for the Wald criterion [6]

$$W_{\min} \leq W,$$

where $W_{\min}$ – the lower limit of the Wald criterion acceptable for decision maker.

In case of existence of strategies S_i , for which $\underline{b}_{i1}$ are in the interval $[W_{\min}, W]$, it is possible to choose a strategy better than S_w . To do this, we determine the increase in payoffs compared to the minimum possible

$$\delta_{ij} = \underline{b}_{ij} - \underline{b}_{i1}.$$

Average payoff increment for i -th strategies will be characterized by the average linear deviation relative to $\underline{b}_{i1}$

$$\langle \delta_i \rangle = \frac{1}{m-1} \sum_{j=2}^m \delta_{ij}. \quad (2)$$

When comparing two strategies, the strategy that provides the greater average gain is more profitable. In the case where there are several strategies $\underline{b}_{i1} \in [W_{\min}, W]$, one should choose a strategy with the maximum $\langle \delta_i \rangle$.

To determine the optimal strategy according to the maximax criterion (optimism criterion), we construct a matrix of the upper boundary values of the intervals and for each strategy S_i , we perform a permutation of payoffs $\bar{a}_{ij}$ by arranging them in non-decreasing order. We will denote this matrix as $\bar{B}$, and its elements as $\bar{b}_{ij}$.

Since the last column in the matrix $\bar{B}$ consists of maximum payoffs, so we write the maximax criterion M in the form of

$$M = \max_i \bar{b}_{im}.$$

The strategy corresponding to the criterion M will be denoted as S_M .

Let us introduce into consideration the admissible interval for the maximax criterion [6]

$$M_{\min} \leq M,$$

where $M_{\min}$ – the lower limit of the criterion acceptable to the decision maker.

For strategies S_i , where $\bar{b}_{im} \in [M_{\min}, M]$, there may be a better choice than S_M . For this purpose, we will determine the decrease in payoffs compared to the maximum possible

$$\Delta_{ij} = \bar{b}_{im} - \bar{b}_{ij}. \quad (3)$$

Average decrease in payoffs for i -th strategies will be described by the average linear deviation relative to $\bar{b}_{im}$

$$\langle A_i \rangle = \frac{1}{m-1} \sum_{j=1}^{m-1} A_{ij}. \quad (4)$$

Of the two strategies, preference should be given to the one with the lower value $\langle A_i \rangle$. If for several strategies $\bar{b}_{im} \in [M_{\min}, M]$, then you should choose a strategy with a minimum $\langle A_i \rangle$.

Example

Let's consider a model example illustrating decision making in the context of interval data. Geophysical studies yielded four possible ore deposit characteristics, which we will identify with nature states. There are three deposit development strategies. The game matrix is as follows:

$$A = \begin{pmatrix} [6.0, 10.1] & [13.6, 15.9] & [12.0, 14.3] & [9.0, 11.2] \\ [8.0, 10.4] & [13.0, 16.0] & [14.0, 15.6] & [6.8, 8.9] \\ [10.0, 12.7] & [7.0, 9.2] & [11.0, 12.5] & [10.0, 11.9] \end{pmatrix}.$$

Here the payoffs are presented in arbitrary units. We will create a matrix of lower boundary values and, after ranking them, we will obtain

$$\underline{B} = \begin{pmatrix} 6.0 & 9.0 & 12.0 & 13.6 \\ 6.8 & 8.0 & 13.0 & 14.0 \\ 7.0_w & 10.0 & 10.0 & 11.0 \end{pmatrix}.$$

The first column of the matrix $\underline{B}$ consists of minimal elements. The maximum of them is $\underline{b}_{31} = 7.0$ (it is provided with an index W), therefore, according to the Wald criterion, the third strategy is optimal. Let's assume that the decision maker is allowed to decrease the payoff by 0.5. Since 6.8 is in the range of 6.5–7.0, let's compare the strategies S_2 and S_3 by average linear increments. Calculations using formula (2) yielded the result $\langle \delta_2 \rangle = 4.87$ and $\langle \delta_3 \rangle = 3.33$, that is $\langle \delta_2 \rangle > \langle \delta_3 \rangle$. Thus, it is rational to choose the second strategy.

If the decision maker is focused on an optimistic result, then we will form a matrix of upper boundary values and, after ranking, we will obtain

$$\bar{B} = \begin{pmatrix} 10.1 & 12.2 & 14.3 & 15.9 \\ 8.9 & 10.4 & 15.6 & 16.0_M \\ 9.2 & 11.9 & 12.5 & 12.7 \end{pmatrix}.$$

According to the maximax criterion $M = \bar{b}_{23} = 16.0$ (with index M), the second strategy is optimal. Let the previous value of decreasing payoff be acceptable, since $15.9 \in [15.5, 16.0]$, then having compared strategies S_1 and S_2 by average linear increments, we obtain, according to formula (4), that $\langle A_1 \rangle = 3.7$ and $\langle A_2 \rangle = 4.37$, that is $\langle A_1 \rangle < \langle A_2 \rangle$. From which it follows that it is rational to choose the first strategy.

4. Conclusions

Under conditions of uncertainty in the probabilities of nature states, the elements of the game matrix are payoffs defined as intervals.

The application of the Wald and maximax criteria, calculated using the lower and upper values of the payoff intervals, respectively, is considered.

If a decision maker accepts a decrease in the criterion within acceptable limits, then it is possible to select a superior strategy compared to the strategy determined based on the criterion.

When using the Wald criterion, the payoff increments are additionally determined for each strategy compared to the minimum possible, and are linearly averaged. When comparing strategies, the one with the larger mean deviation is considered superior.

When using the maximax criterion, the decreases in payoffs compared to the maximum possible are additionally determined and linearly averaged. When comparing strategies, the one with the smaller mean deviation is considered superior.

Conflict of interest

Authors state no conflict of interest.

REFERENCES

1. Taha, Hamdi A. (2017), *Operations Research. An Introduction*, 10th Edition, Pearson Education Limited, Harlow, Essex, England, available at: <https://zalamasyah.staff.unja.ac.id/wp-content/uploads/sites/286/2019/11/9-Operations-Research-An-Introduction-10th-Ed.-Hamdy-A-Taha.pdf> (Accessed 05 December 2025).
2. Shiyan, A.A. (2009), *Teoriia ihor: osnovy ta zastosuvannia v ekonomitsi ta menedzhmenti* [Game Theory: Fundamentals and Applications in Economics and Management], Vinnytsia National Technical University (VNTU), Vinnytsia, Ukraine, available at: https://duikt.edu.ua/uploads/1465_75966459.pdf (Accessed 05 December 2025).
3. Petrushenko, M.M. (2012), "Economic "games against nature": a decision-making model in the field of environmental conflict management", *Business Inform*, no. 4, pp. 130–132, available at: http://www.irbis-nbuv.gov.ua/cgi-bin/irbis_nbuv/cgiirbis_64.exe?I21DBN=LINK&P21DBN=UJRN&Z21ID=&S21REF=10&S21CNR=20&S21STN=1&S21FMT=ASP_meta&C21COM=S&S21P03=FILA=&S21STR=binf_2012_4_41 (Accessed 05 December 2025).
4. Kaluski, J. (2011), "Application of game theory in planning and resource management of a mining enterprise", *Computing*, vol. 10, no. 2, pp. 172–176, available at: <http://dspace.tneu.edu.ua/handle/316497/32372> (Accessed 05 December 2025).
5. Bulat, A.F., Bunko, T.V., Kokoulin, I.E. and Yashchenko, I.O. (2017), "Criteria for using the risk matrix when improving the production and occupational health and safety management system in coal mines", *Geo-Technical Mechanics*, vol. 133, pp. 228–238, available at: <http://inas.nbuv.gov.ua/article/UJRN-0000835304> (Accessed 05 December 2025).
6. Lapshyn, Ye., Molchanov, P., Blyuss, B. and Osadcha, H. (2019), "Improving the criteria for choosing the strategies in management by geotechnical systems", *E3S Web of Conferences*, 109, 00048, <https://doi.org/10.1051/e3sconf/201910900048>
7. Shubik, M. (2011), "The Present and Future of Game Theory", *SSRN Electronic Journal*, 57(01), 1250001–1, <https://doi.org/10.2139/ssrn.1883520>
8. Ulansky, V. and Raza, A. (2021), "Generalization of minimax and maximin criteria in a game against nature for the case of a partial a priori uncertainty", *Heliyon*, vol. 7, iss. 7, pp. 1–6, <https://doi.org/10.1016/j.heliyon.2021.e07498>
9. Kushlyk-Dyvul'ska, O.I. and Kushlyk, B.R. (2014), *Osnovy teorii pryiniattia rishen* [Fundamentals of Decision Making Theory], NTUU KPI, Kyiv, Ukraine, available at: <https://ela.kpi.ua/server/api/core/bitstreams/f9d59169-7c45-4661-8165-6931d004ca62/content> (Accessed 05 December 2025).

10. Reshetylo, V.P. and Fedotova, Y.V. (2020), "Uncertainty and risk: the relationship of concepts and the specifics of decision-making", *Economics and Enterprise Management*, no. 3 (77), pp. 149–154. <https://doi.org/10.32782/2520-2200/2020-3-41>
11. Nikitina, L. and Yatsenko, I. (2023), *Modeli ta metody pryiniattia risheh* [Decision-making Models and Methods], NTU "KhPI", Kharkiv, Ukraine, available at: <https://repository.kpi.kharkov.ua/handle/KhPI-Press/65270> (Accessed 05 December 2025).
12. Reva, O.M., Kamyshin, V.V., Nevynitsyn, A.M., Borsuk, S.P. and Shulgin, V.A. (2020), "Application of classical decision-making criteria to determine the risk-uncertainty of air traffic controllers' preference systems on the hazards of characteristic errors", *Science, Technologies, Innovations*, no. 2, pp. 54–67, <http://doi.org/10.35668/2520-6524-2020-2-07>
13. Loganathan, C. and Annie Christi, M.S. (2012), "Fuzzy Game Value of the Interval Matrix", *International Journal of Engineering Research and Applications*, vol. 2, iss. 5, pp. 250–255, available at: https://www.academia.edu/11474219/Fuzzy_Game_Value_of_the_Interval_Matrix (Accessed 05 December 2025).
14. Hwang, Y. and Liao, Y. (2025), "Resolution Under Interval Uncertainty", *Mathematics*, No 5, 762, <https://doi.org/10.3390/math13050762>
15. Romanenkov, Yu., Kosenko, V., Lobach, O., Grinchenko, E. and Grinchenko, M. (2019), "The Method for Ranking Quasi-Optimal Alternatives in Interval Game Models Against Nature", *Proceedings of the 3rd International Conference on Computational Linguistics and Intelligent Systems (COLINS-2019)*, Volume I, Kharkiv, Ukraine, pp. 334–343, available at: https://www.researchgate.net/publication/333488747_The_Method_for_Ranking_Quasi-Optimal_Alternatives_in_Interval_Game_Models_Against_Nature (Accessed 05 December 2025).
16. Ignashkina, T.B., Goncharuk, O.V. and Naydovska, A.O. (2023), *Obgruntuvannia hospodarskykh risheh i otsiniuvannia ryzykiv* [Justification of business decisions and risk assessment, part 2], Ukrainian State University of Science and Technology, Dnipro, Ukraine, available at: <https://crust.ust.edu.ua/server/api/core/bitstreams/8e6b11f3-6321-413d-98fa-f03f01ae1d8d/content> (Accessed 05 December 2025).
17. Kulyk, Y.M. (2012), "Formation of a risk management model to ensure the reliability of the enterprise's logistics system", *Economy and State*, no. 5, pp. 74–78, available at: <http://www.economy.in.ua/index.php?op=1&z=1826&i=18> (Accessed 05 December 2025).
18. Gaspars-Wieloch, H. (2014), "Modifications of the maximin joy criterion for decision making under uncertainty", *Quantitative Methods in Economics*, vol. XV, no. 2, pp. 84–93, available at: <https://scispace.com/pdf/modifications-of-the-maximin-joy-criterion-for-decision-37vc56tnc0.pdf> (Accessed 05 December 2025).
19. Hreaveargs, G.I. (2002), "Interval Analysis in MATLAB", *Numerical Analysis Report*, available at: <https://eprints.maths.manchester.ac.uk/1204/1/> (Accessed 05 December 2025).

ПРИЙНЯТТЯ РІШЕНЬ У ГІРНИЧІЙ СПРАВІ НА ОСНОВІ ТЕОРІЇ ІГР З ПРИРОДОЮ ПРИ ІНТЕРВАЛЬНИХ ДАНИХ

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Анотація. Необхідність прийняття рішень є у всіх сферах діяльності (соціологія, медицина, економіка, військова справа та інших). Вкрай важливо це й у гірничій справі. Для прийняття рішень широко використовується теорія ігор. Теорія ігор – ефективний інструмент прийняття рішень під час планування та управління гірничодобувними підприємствами. Розглянуто завдання про прийняття рішень за умов невизначеності, під якою розуміється відсутність чи брак інформації. І тут доцільно використання математичної моделі «Гра з природою». У ній свідомий «гравець» – особа, яка приймає рішення (ОПР), а «природа» – навколишнє середовище. При цьому відсутня інформація про ймовірність стану природи. Здебільшого аналізуються ігри з детермінованими даними (матрицями виграшів). Проте, практично їх неможливо визначити точно. Це зумовлено багатьма причинами. Одна з них – дані реально не визначаються повністю стратегіями учасників гри, а також залежать від багатьох інших факторів. Особливість гри з інтервальними даними у тому, що виграші не задані точно. Для них відомі лише межі можливих значень, верхня і нижня, чи, що еквівалентно, дано лише інтервали, у яких можуть бути значення виграшів. Матриця, в якій виграшам приписані не конкретні числа, а інтервали можливих значень, точніше відповідає реальності Роботи, в яких розглянуті ігри з інтервальними даними в основному присвячені виявленню існування сідлової точки та розробки методів її пошуку. Для вибору оптимальної стратегії найчастіше застосовують такі класичні критерії: максимінний Вальда та максимуму. Однак вони не враховують інтервальні розподіли всіх виграшів за різних станів природи. Мета роботи – удосконалення застосування критеріїв Вальда та максимуму шляхом урахування впливу всіх станів природи на вибір стратегії прийняття рішення при інтервальних даних. Проаналізовано застосування критеріїв Вальда та максимуму, які обчислювалися відповідно за нижніми та верхніми значеннями інтервалів виграшів. Якщо для ОПР прийнятне зменшення критерію в допустимих межах, то можливий вибір кращої стратегії порівняно зі стратегією, визначеною на основі критерію. При використанні критерію Вальда для кожної стратегії додатково визначаються збільшення виграшів у порівнянні з мінімально можливими і проводиться їхнє лінійне усереднення. При порівнянні стратегій вигідніша та, що забезпечує більше середнє значення відхилень. Якщо ж застосовується критерій максимуму, то додатково визначаються зменшення виграшів у порівнянні з максимально можливими і проводиться їхнє лінійне усереднення. При порівнянні стратегій вигідніша та, яка забезпечує менше середнє значення відхилень.

Ключові слова: гра, природа, невизначеність, інтервал, критерії, середнє відхилення.