

Yevgen Semenenko^{1,*}, <https://orcid.org/0000-0001-8707-3648>**Borys Blyuss**¹, <https://orcid.org/0000-0003-3077-1220>**Oleksandr Khaminich**², <https://orcid.org/0000-0002-2621-8206>**Raissa Zhanakova**³, <https://orcid.org/0000-0003-0845-8449>¹M.S. Poliakov Institute of Geotechnical Mechanics of the National Academy of Sciences of Ukraine, Dnipro, Ukraine²Oles Honchar Dnipro National University (DNU) under the Ministry of Education and Science of Ukraine, Dnipro, Ukraine³Satbayev University, Almaty, Kazakhstan*Corresponding author e-mail: evs_igtm@i.ua

INCREASING THE ACCURACY OF DETERMINING THE CRITICAL SPEED OF HYDROTRANSPORTATION USING GRAVITY THEORY

Abstract. The efficiency of operation and the energy intensity of the operating modes of pressure hydraulic transport systems at mining enterprises largely depend on the accuracy of calculating the critical velocity of hydrotransportation, which determines the minimum permissible velocity of the hydro-mixture flow, as well as its possible concentration and density. Most formulas used to calculate the critical velocity of hydrotransportation contain empirical constants that vary within certain limits. There are no precise recommendations for selecting the values of these constants, which leads to errors in determining the critical velocity of hydrotransportation and, overall, either to an overestimation of the energy intensity of the hydrotransport process if this value is significantly overestimated, or to inefficient operating modes if it is underestimated. The largest group of methods for determining the critical velocity of hydrotransportation is based on the gravitational theory of M.A. Velikanov and uses the principle of proportionality between the additional hydraulic gradient and its total value to determine the onset of the critical regime. The coefficient of this proportionality is an empirical value and is referred to as the Velikanov constant. The author of the gravitational theory of hydro-mixture flow did not provide any recommendations for determining this constant, while researchers who developed formulas for calculating the critical velocity of hydrotransportation using this approach embedded its value within other experimental constants that account for the influence of many factors. It should be noted that the error arising from the use of these formulas was for a long time compensated by overestimating the critical velocity of hydrotransportation. Under the economic conditions of the former USSR, when the cost of electrical energy was low, this did not affect the competitiveness of domestic enterprises. However, under new economic conditions, when the country's mining enterprises compete on the global market with more developed competitors, the requirements for the energy efficiency of hydraulic transport systems are increasing, which necessitates a more accurate determination of the critical velocity of hydrotransportation. This paper, for the first time, establishes the dependence of coefficients in the formulas for determining the critical velocity of hydrotransportation on the Velikanov constant and proposes a method for determining this parameter depending on the characteristics of solid-phase particles in the hydro-mixture and the critical velocity of hydrotransportation.

Keywords: hydro-mixture, hydrotransport, critical velocity, gravitational theory.

1. Introduction

Domestic mining enterprises ensure a guaranteed share of the country's export potential and, in the near future, will have to face fierce competition on the global market from more developed competitors [1–8]. The production cost of domestic mining enterprises largely depends on the efficiency of operation and the energy intensity of the operating modes of their technological units, one of the largest of which is the transportation of primary placer materials and their processing waste by means of pressure hydraulic transport. The efficiency of operation and the energy intensity of the operating modes of such pressure hydraulic transport systems at mining enterprises significantly depend on the accuracy of calculating the critical velocity of hydraulic transportation (CVHT), which determines the minimum permissible velocity of the hydromixture flow, as well as its possible concentration and density. Most

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formulas used to calculate the CVHT contain empirical constants that vary within certain limits. There are no precise recommendations for selecting the values of these constants, which leads to errors in determining the CVHT and, overall, either to an overestimation of the energy intensity of the hydraulic transport process if this value is significantly overestimated, or to inefficient operating modes if it is underestimated.

In the territories of the former USSR, 17 methods for calculating the CVHT were known, most of which namely seven are based on the gravitational theory of M.A. Velikanov. All these methods for calculating the CVHT include empirical coefficients whose values vary within the range of 16% to 50% [9]. Considering that the permissible range of the CVHT variations is regulated at no more than 30% [10, 11], such an error in determining the coefficients during CVHT calculations almost fails to meet the required accuracy of this parameter. It should be noted that the error arising from the use of these formulas was for a long time compensated by overestimating the CVHT. Under the economic conditions of the former USSR, when the cost of electrical energy was low, this did not affect the competitiveness of domestic enterprises. However, under new economic conditions, when the country's mining enterprises compete on the global market with more advanced competitors, the requirements for the energy efficiency of hydraulic transport systems are increasing, which necessitates a more accurate determination of the CVHT [11–16].

Most modern domestic studies bypass this issue, focusing instead on problems such as coagulation of the solid phase, the use of polymer pipelines, or the transportation of high-concentration hydro-mixtures [17–22]. Foreign researchers also tend to focus on other issues, such as the influence of the physical properties of the solid phase [12, 14–16], the use of pipelines with different cross-sectional shapes and route profiles [13, 24–26], the merging of two hydro-mixture flows [23], processes of changes in the structure of the bottom layer and its granulometric composition [27, 28], and the characteristics of high-concentration hydromixture flows [29]. For the most part, foreign researchers do not use the gravitational theory at all, determining the CVHT through the shear stress of solid particles against the flow bed [23, 30–33], although the results of some studies can be considered as partial confirmation of this theoretical model of hydromixture flow [25, 30, 34–40].

Thus, the aim of this study is to determine the dependence of empirical constants in the formulas for calculating the CVHT on the M.A. Velikanov constant, and to derive a formula for calculating the value of M.A. Velikanov constant itself.

2. Methods

The largest group of methods for determining the critical velocity of hydrotransportation (CVHT), known in the former USSR, is based on the gravitational theory of M.A. Velikanov and uses the principle of proportionality between the additional hydraulic gradient and its total value to determine the onset of the critical regime [9, 11, 21, 22]:

$$\frac{i_{cm} - i_{cm}^0}{i_{cm}} = K, \quad (1)$$

where i_{cm} – hydraulic gradient of the hydraulic mixture during flow with CVHT, m water column/m; i_{cm}^0 – hydraulic gradient of the fluid during flow with CVHT, m water column/m; K – M.A. Velikanov constant.

The proportionality coefficient in formula (1) is an empirical value and is called the M.A. Velikanov constant. The author of the gravitational theory of hydromix flow left no recommendations for determining this constant, and scientists who developed formulas for calculating the CVHT based on this principle concealed its value within other experimental constants that account for the influence of many factors.

One of the first methods for calculating the CVHT, based on M.A. Velikanov gravitational theory, was developed for the conditions of pressurized hydraulic transport of polydisperse coal [9, 11, 21, 22]:

$$V_{cm} = c_0 \sqrt{gD} \sqrt[3]{\frac{wR_2}{\sqrt{d_{gs}}}} P + c_1 \sqrt{fgDPR_3}; \quad P = \frac{(\rho_s - 1)(1 - SR_1)}{(1 + (\rho_s - 1)SR_1)} S; \quad (2)$$

$$i = i_0 + P \left(R_1 i_0 + cR_2 \frac{w}{V} \sqrt{\frac{D}{d_{gs}}} + fR_3 \right); \quad i_0 = \frac{\lambda V^2}{2gD}; \quad (3)$$

where V_{cm} – CVHT, m/s; c_0, c_1, c – empirical constants ($c_0 = 1.8 \div 2.1$, $c_1 = 6 \div 7$ and $c = 0.3 \div 0.45$); D – pipe diameter, m; g – acceleration due to gravity, m/s²; w – the terminal velocity of fine-fraction particles, i.e., particles ranging in size from 0.1 mm to 2 mm, m/s; S – volume concentration of the aqueous mixture, units; d_{gs} – average particle size of the fine fractions, m; ρ_s – the relative bulk density of the material being transported; R_1, R_2, R_3 – the bulk fraction of fine, small, and lumpy particles in the particle size distribution of the solid phase; f – the generalized coefficient of friction between the particles of the coarse fractions and the inner surface of the pipeline; i – hydraulic head of the hydraulic fluid, m water column/m; i_0 – hydraulic head, m water column/m; λ – coefficient of hydraulic friction; V – the average flow velocity of the hydraulic mixture, m/s.

Upon examining formulas (1)–(3), it becomes clear that the empirical constants c_0 and c_1 must depend on M.A. Velikanov constant; they must incorporate this quantity in some form. To determine this dependence, it is necessary to consider formula (1) together with the first formula from (3), derive a formula for calculating the average flow velocity, and compare it with the first formula from (2).

3. Theoretical part

The analysis of the first formula in (2) indicates that the constant is dimensional and can be represented in the following form

$$c_0 = \frac{c'_0}{\sqrt[6]{g}},$$

therefore, instead of the first formula from (2), the following expression should be used

$$V_{cm} = c'_0 \sqrt{gD} \sqrt[3]{\frac{wR_2}{\sqrt{gd_{gs}}} P}, \quad (4)$$

where c'_0 – dimensionless empirical constants ($c'_0 = 2.63 \div 3.07$).

Thus, assuming that the transported material contains only fine particles,

$$i_{cm} = i_{cm}^0 (1 + PR_1), \quad (5)$$

equation (1) does not allow us to determine the CVHT, since the numerator and denominator of the fraction on the left-hand side of the condition are directly proportional to the hydraulic head of the fluid, which decreases, and we obtain the first estimate of the value of M.A. Velikanov constant:

$$K' = \frac{PR_1}{1 + PR_1}, \quad (6)$$

where K' – the value of M.A. Velikanov constant for a material consisting solely of fine grained particles.

Assuming that the transported material contains no lumpy fractions, and considering the hydraulic friction resistance coefficient to be a constant with respect to the velocity of the hydraulic mixture, as prescribed by the methodology [9, 11, 21, 22], by combining the first formula from (3) with equation (1), after performing the appropriate transformations, we obtain the following expression for determining CVHT

$$\frac{V_{cm}}{\sqrt{gD}} = \sqrt[3]{\frac{2c}{\lambda_{cm}} \frac{PR_2}{\frac{K}{1-K} - PR_1} \frac{w}{\sqrt{gd_{gs}}}}, \quad (7)$$

where λ_{cm} – the coefficient of hydraulic friction resistance in the critical regime of hydraulic transport.

By comparing equation (7) with equation (4), it is easy to see that

$$c'_0 = \frac{1}{\sqrt[3]{\frac{K}{1-K} - PR_1}} \sqrt[3]{\frac{2c}{\lambda_{cm}}}. \quad (8)$$

In the general case, when the granulometric composition of the solid phase contains fine, small, and coarse particles, the CVHT is determined depending on the value of the governing parameter of the properties of the transported material [9, 11, 21, 22]

$$\frac{V_{cm}}{\sqrt{gD}} = \frac{1}{\sqrt[3]{\frac{K}{1-K} - PR_1}} \sqrt[3]{\frac{2}{\lambda_{cm}}} \sqrt[3]{\frac{w}{\sqrt{gd}}} x; \quad (9)$$

$$x = \begin{cases} \sqrt[3]{\frac{1}{2}} \left[\sqrt[3]{1 + \sqrt{1 - \frac{4\theta^3}{27}}} + \sqrt[3]{1 - \sqrt{1 - \frac{4\theta^3}{27}}} \right], & \theta \leq \sqrt[3]{\frac{27}{4}} \\ 2\sqrt{\frac{\theta}{3}} \cos\left(\frac{1}{3} \arccos\left(\sqrt{\frac{27}{4\theta^3}}\right)\right), & \theta > \sqrt[3]{\frac{27}{4}} \end{cases} \quad (10)$$

$$\theta = \sqrt[3]{\frac{2}{c_1^3 \lambda_{cm}}} \frac{R_3}{R_2} \frac{f}{\left(\frac{w}{\sqrt{gd}}\right)^{\frac{2}{3}}} \frac{1}{\sqrt[3]{\frac{K}{1-K} - PR_1}}, \quad (11)$$

where θ – a key parameter determining the properties of the material being transported.

It follows from equation (1) that the M.A. Velikanov constant does not exceed unity, and using equations (6) through (9) and (11), it is easy to prove that

$$\frac{K'}{1-K'} = PR_1,$$

and to establish a lower bound for the M.A. Velikanov constant

$$K' < K < 1. \quad (12)$$

From the comparison of formulas (6) and (12), it follows that the value of the M.A. Velikanov constant for a material consisting only of fine particles represents the minimum permissible value of the M.A. Velikanov constant when transporting polydisperse material.

Condition (1) imposes a certain restriction on the ratio between the hydraulic gradient of the hydromix and the hydraulic gradient of the liquid in the critical regime

$$i_{cm} = \frac{i_{cm}^0}{1-K}, \quad (13)$$

which, at first glance, is not satisfied by the first formula in (3). However, if we consider the first formulas together with (2) and (3), then, after performing the appropriate transformations, we obtain

$$i = i_0(1 + PR_1) + \frac{c}{c_0'^2} \frac{V_{cm}^3}{gDV}. \quad (14)$$

If condition (1) is applied to formula (14), it is not difficult to show that the value of the M.A. Velikanov constant can be calculated as follows

$$K = 1 - \frac{1}{1 + PR_1 + \frac{2c}{\lambda_{cm}c_0'^2}}. \quad (15)$$

From expression (15), it follows that the value of the M.A. Velikanov constant calculated using this formula automatically satisfies condition (12), since

$$K' = 1 - \frac{1}{1 + PR_1}. \quad (16)$$

Using the formula, it is easy to prove that the ratio in the sub-root expression in formulas (6)–(9) and (11) will be

$$\frac{K}{1-K} = PR_1 + \frac{2c}{\lambda_{cm}c_0'^2},$$

and instead of formulas (7), (9)–(11), the following relationships should be used.

Provided that there are no lumpy fractions in the transported material,

$$\frac{V'_{cm}}{\sqrt{gD}} = \sqrt[3]{c_0'^2} \sqrt[3]{PR_2 \frac{w}{\sqrt{gd_{gs}}}}, \quad (17)$$

and in the general case, when the particle size distribution of the solid phase contains particles of fine, small, and lumpy fractions, if $\Gamma \leq 1$, then you should use

$$\frac{V'_{cm}}{\sqrt{gD}} = \sqrt[3]{\frac{c_0'^2}{2c}} \sqrt[3]{\frac{w}{\sqrt{gd}}} \left(\sqrt[3]{1 + \sqrt{1 - \Gamma^3}} + \sqrt[3]{1 - \sqrt{1 - \Gamma^3}} \right), \quad (18)$$

and in the event that $\Gamma > 1$, then, to determine the CVHT, you need to use

$$\frac{V'_{cm}}{\sqrt{gD}} = \frac{2c_0'}{\sqrt{3cc_1}} \sqrt{\frac{R_3}{R_2}} \sqrt{f} \cos \left(\frac{1}{3} \arccos \left(\frac{1}{\sqrt{\Gamma^3}} \right) \right); \quad (19)$$

$$\Gamma = \frac{R_3}{R_2} \frac{f}{\Phi} \sqrt[3]{\frac{gd}{w^2}}; \quad \Phi = \sqrt[3]{\frac{27cc_1^3}{4c_0'^2}},$$

where V'_{cm} – CVHT, calculated using the refined method, m/s; Γ – Smoldirev number; Φ – a complex empirical constant.

Thus, formulas (7), (9), and (11) establish the dependencies of the empirical constants in the formulas for determining the CVHT on M.A. Velikanov constant, while formulas (12), (15), and (16) allow for the calculation of the value of M.A. Velikanov constant and the limits of its variation. At the same time, a combined analysis of these formulas yielded formulas (17)–(19), according to which the CVHT is calculated using sets of empirical constants.

4. Results and discussion

To study formulas (4)–(19), it is reasonable to rewrite them in a more convenient form. Thus, for the minimum permissible value and the current value of the M.A. Velikanov constant, the following can be proposed:

$$K' = 1 - \frac{1}{1 + \frac{Ar(1-s)}{(1+Ars)s}}; \quad Ar = \rho_s - 1; \quad s = SR_1, \quad (20)$$

$$K = 1 - \frac{1}{\frac{1}{1-K'} + z}; \quad z = \frac{6.47c}{c_0'^2} (\lg Re_{cm} - 1)^2; \quad Re_{cm} = \frac{V_{cm}D}{\nu}, \quad (21)$$

where Ar – Archimedes parameter; s – volume concentration of fine particles in the hydraulic mixture, units; z – turbulence parameter; Re_{cm} – the Reynolds number for the critical regime of hydrodynamic transport.

Formula (20) allows determining the limits of variation of the minimum permissible value of the M.A. Velikanov constant during the transport of polydisperse material based on the volume concentration of fine fractions in the hydromix at various values of the Archimedes parameter (Figure 1). From the graphical dependencies shown in Figure 1, it can be seen that the minimum permissible values of the M.A. Velikanov constant are limited to the following interval:

$$0 \leq K' \leq 0.301,$$

which allows determining the ranges of variation for the term appearing first in the formulas (21)

$$1 \leq \frac{1}{1-K'} \leq 1.429.$$

The second of the formulas (21) determines the ranges of variation for the

turbulence influence parameter as a function of the Reynolds number for the critical regime of hydrodynamic transport (Figure 2).

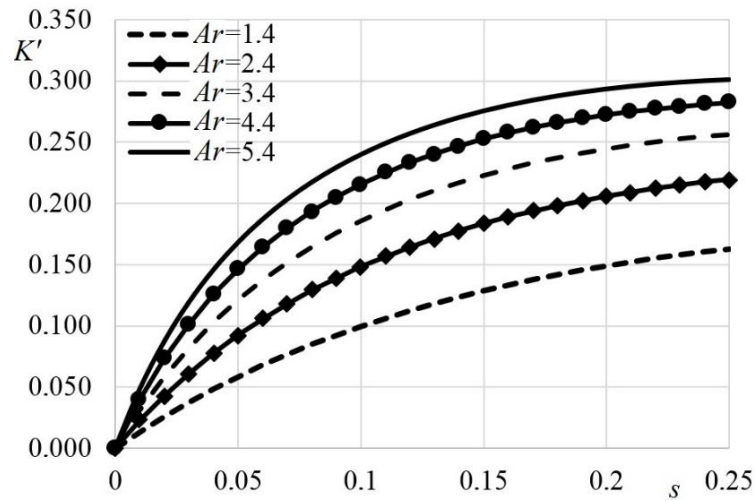


Figure 1 – Dependence of the minimum permissible value of the M.A. Velikanov constant during transportation of polydisperse material on the volumetric concentration of fine fractions in the hydro-mixture for different values of the Archimedes parameter

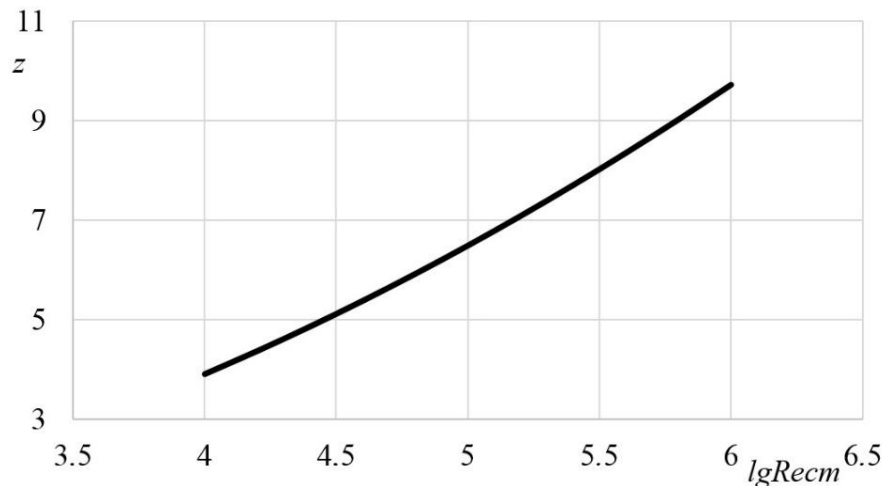


Figure 2 – Dependence of the turbulence influence parameter on the Reynolds number for the critical hydrotransport regime

Calculating the ranges of variation for these two parameters allows us to investigate the dependence of the value of the M.A. Velikanov constant on the Reynolds number for the critical regime of hydrotransport at different values of its minimum permissible value (Figure 3), from which a weak dependence of the value of the M.A. Velikanov constant on its minimum permissible value, which allows us to propose, when performing calculations with engineering accuracy, to calculate the value of M.A. Velikanov constant using only the parameter z .

Based on the first of the formulas (21), for this case, we can propose an expression obtained analytically:

$$K = 1 - \frac{1}{z}, \quad (22)$$

but numerical studies of the dependencies shown in Figure 3 indicate the possibility of approximating them with power and logarithmic functions, with an approximation accuracy of $R^2 = 0.9896$ and $R^2 = 0.9928$, accordingly:

$$K = \left(\frac{z}{19.163} \right)^{0.1326}; \quad (23)$$

$$K = 0.114 \ln \left(\frac{z}{0.003} \right). \quad (24)$$

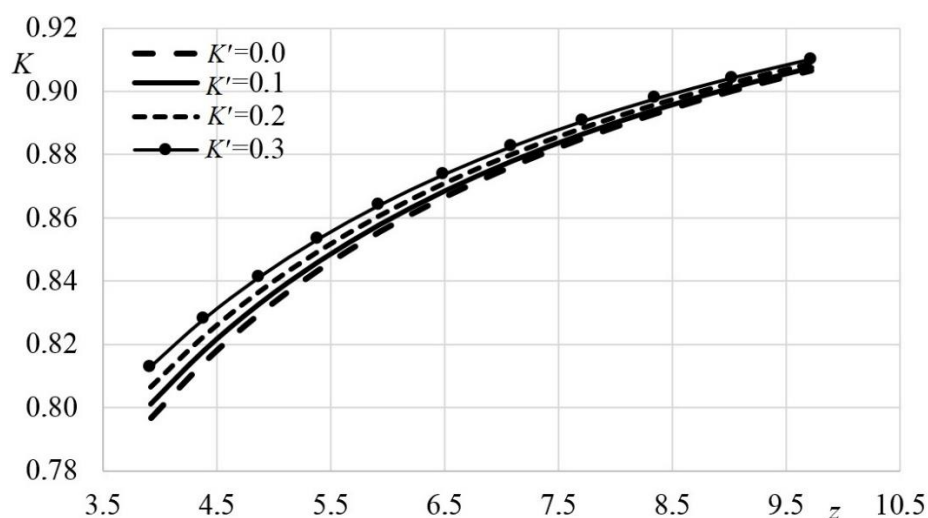


Figure 3 – Dependence of the M.A. Velikanov constant on the Reynolds number for the critical hydrotransport regime at different values of its minimum permissible value

A comparison of the accuracy of calculations of the value of the M.A. Velikanov constant using formulas (22)–(24), within the corresponding range of Reynolds numbers, was performed relative to its value generalized by the density of solid-phase particles (Table 1). The results of this comparison indicate the potential for using the power function, i.e., formula (23), for calculations with engineering accuracy. Formula (13), which reflects the relationship between the hydraulic gradient of the hydromix and the hydraulic gradient of the liquid in the critical regime, taking into account the data shown in Figure 3, should be rewritten in the following form (Figure 4):

$$j = \frac{1}{1 - K'} + z; \quad j = \frac{i_{cm}}{i_0}, \quad (25)$$

where j – the multiple of the hydraulic gradient in critical flow.

Processing of the calculation results presented in Figure 4 shows that the dependence of the hydraulic gradient ratio in the critical regime generalized with respect to the minimum permissible value of the M.A. Velikanov constant on the turbulence influence parameter can be approximated with high accuracy by a linear function:

$$j = 1.197 + z.$$

Table 1 – Comparison of the accuracy of calculations of the value of K using the formulas (22)–(24)

$lgRe_{cm}$	General meaning K	Calculation using the formula			Relative error, %		
		(22)	(23)	(24)	(22)	(23)	(24)
4.0	0.804	0.926	0.810	0.819	8.00	-0.72	-1.75
4.2	0.821	0.940	0.822	0.831	6.33	-0.21	-1.30
4.4	0.835	0.952	0.834	0.843	5.08	0.14	-0.99
4.6	0.848	0.960	0.845	0.855	4.14	0.34	-0.81
4.8	0.860	0.967	0.856	0.866	3.40	0.43	-0.73
5.0	0.870	0.973	0.866	0.876	2.83	0.42	-0.73
5.2	0.879	0.977	0.876	0.886	2.37	0.32	-0.79
5.4	0.888	0.980	0.886	0.896	2.00	0.16	-0.91
5.6	0.895	0.983	0.896	0.905	1.70	-0.05	-1.08
5.8	0.902	0.986	0.905	0.914	1.46	-0.31	-1.28
6.0	0.908	0.988	0.914	0.922	1.26	-0.61	-1.51

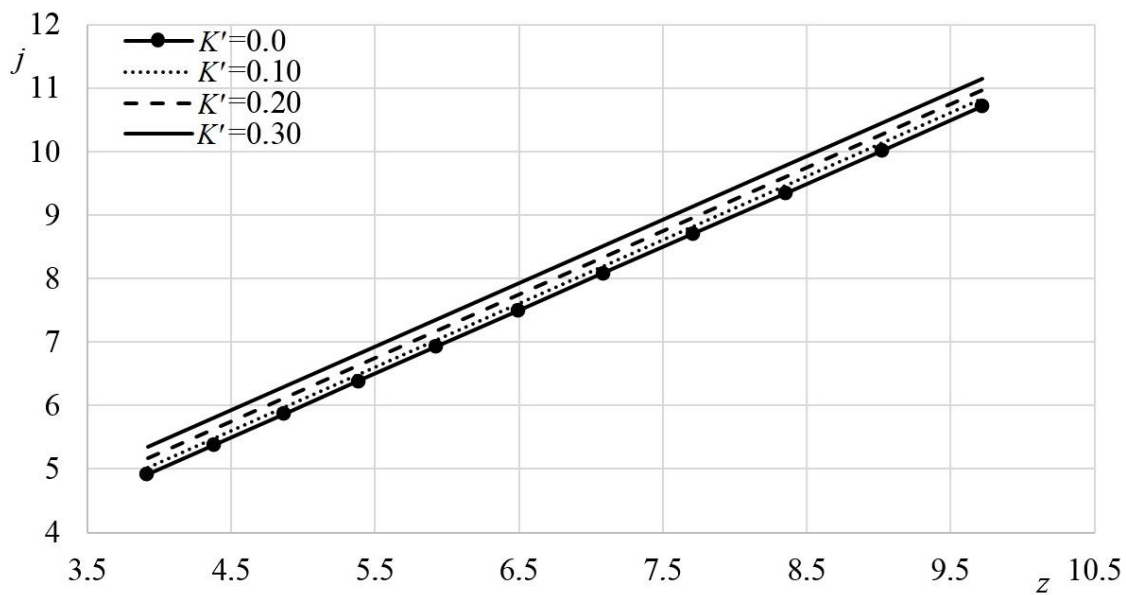


Figure 4 – Dependence of the hydraulic gradient ratio in the critical regime on the turbulence influence parameter at different values of the minimum permissible M.A. Velikanov constant

To investigate the effect of the factor accounting for the dependence of the M.A. Velikanov constant on the parameters of the solid phase of the hydro-mixture and the flow characteristics, it is proposed to use the CVHT ratios calculated using the old method and the refined method:

$$\Lambda = \frac{V_{cm}}{V'_{cm}}, \quad (26)$$

which, provided that the transported material contains no lumpy fractions, yields the following estimate

$$\Lambda = \sqrt[3]{c'_0}, \quad (27)$$

and in the general case, when the particle size distribution of the solid phase contains particles of fine, small, and lumpy fractions, at $\Gamma \leq 1$,

$$\Lambda = \sqrt[3]{\frac{2c}{c'_0} \frac{\left(1 + \frac{\sqrt{c_1^3}}{\sqrt[3]{c'_0{}^4}} \sqrt[6]{\frac{27c}{4}} \sqrt{\Gamma} \sqrt[6]{r}\right)}{\sqrt[3]{1 + \sqrt{1 - \Gamma^3}} + \sqrt[3]{1 - \sqrt{1 - \Gamma^3}}}} \sqrt[3]{r}, \quad (28)$$

and in the event that $\Gamma > 1$:

$$\Lambda = \frac{\sqrt{3cc_1^3}}{2c'_0} \frac{\left(1 + \frac{\sqrt[3]{c'_0{}^2}}{\sqrt{c_1}} \sqrt[6]{\frac{27c}{4}} \frac{1}{\sqrt{\Gamma} \sqrt[6]{r}}\right)}{\cos\left(\frac{1}{3} \arccos\left(\frac{1}{\sqrt{\Gamma^3}}\right)\right)} \sqrt{r}; \quad (29)$$

$$r = \frac{Ar(1-s)s}{(1+Ars)} e; \quad e = \frac{R_2}{R_1},$$

where Λ – the CVHT ratio; r – the concentration coefficient; e – the non-uniformity of the particle size distribution of the transported material.

For the convenience of further calculations, it is necessary to determine the generalized values and ranges of variation for the sets of empirical constants used in formulas (14)–(29) (Table 2).

Table 2 shows that in the absence of lumpy fractions in the transported material, the CVHT ratios calculated using the old method and the refined method range from 1.381 to 1.454. To calculate the CVHT ratio in the case of transporting a polydisperse material containing all three fractions, a study was conducted on the dependence of the limiting values of the concentration coefficient on the volume concentration of fine fractions in the hydromix (Figure 5). Taking this into account, calculations were performed using the formulas (28) and (29), (Figures 6, 7).

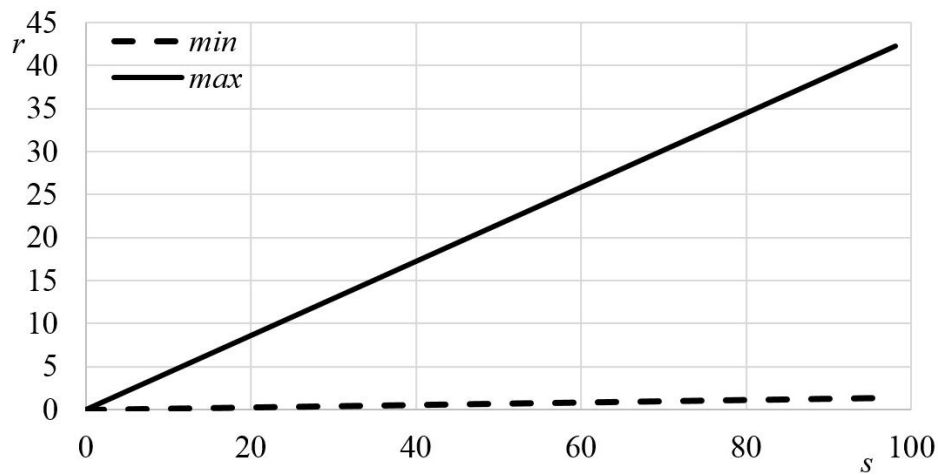


Figure 5 – Dependence of the limiting values of the concentration coefficient on the volumetric concentration of fine fractions in the hydromixture

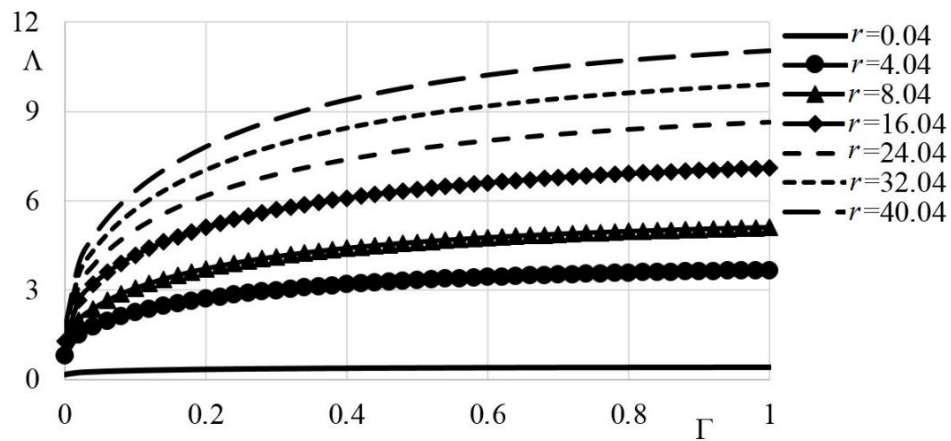


Figure 6 – Dependence of the CVHT ratio on the Smoldyrev number for different values of the concentration coefficient in the case where the solid phase of the hydro-mixture does not contain coarse particles at $\Gamma \leq 1$

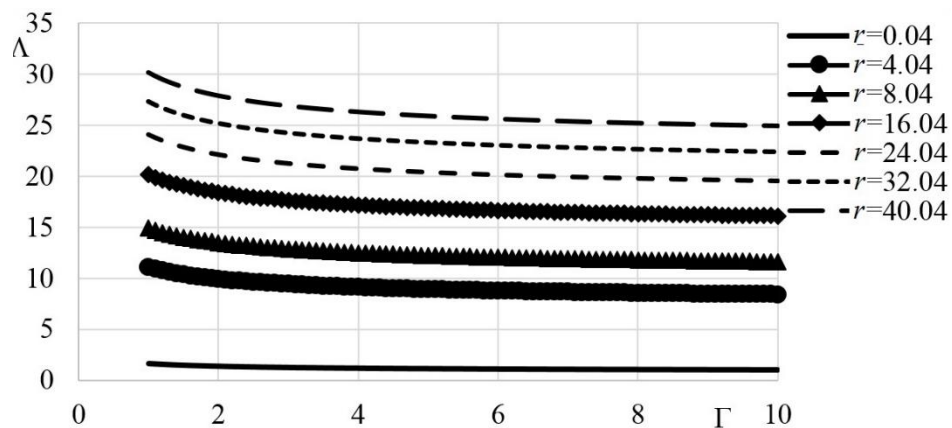


Figure 7 – Dependence of the CVHT ratio on the Smoldyrev number for different values of the concentration coefficient in the case where the solid phase of the hydro-mixture contains coarse particles at $\Gamma > 1$

Table 2 – Values of sets of empirical constants

Set of empirical constants	Value			Interval width	
	minimum	maximum	average	absolute	relative
$2c/c_0'^2$	0.064	0.130	0.097	0.066	0.685
$\sqrt[3]{c_0'^2}$	1.907	2.113	2.010	0.206	0.103
$\sqrt[3]{c_0'^2/(2c)}$	1.975	2.506	2.241	0.531	0.237
$2c_0'/\sqrt{3cc_1}$	1.713	2.644	2.179	0.931	0.427
$\sqrt[3]{27cc_1^3/(4c_0'^2)}$	3.592	5.316	4.454	1.724	0.387
$\sqrt[3]{c_0'}$	1.381	1.454	1.417	0.073	0.051
$\sqrt[3]{2c/c_0'}$	0.580	0.699	0.640	0.119	0.186
$\sqrt{c_1^3\sqrt[6]{27c}}/\sqrt[3]{c_0'^4}/\sqrt[6]{4}$	3.701	6.128	4.915	2.427	0.494
$\sqrt{3cc_1^3}/(2c_0')$	2.269	4.085	3.177	1.816	0.572
$\sqrt[3]{c_0'^2\sqrt[6]{27c}}/\sqrt{c_1}/\sqrt[6]{4}$	0.811	1.038	0.925	0.228	0.246

5. Conclusions

Thus, as a result of improving the gravitational theory of M.A. Velikanov, an analytical solution of the equation for determining the critical velocity of hydrotransportation (CVHT) has been obtained for the first time for various cases of the granulometric composition of the solid phase of the hydro-mixture, taking into account the dependence of the value of this constant on the CVHT.

It has been established that the value of the M.A. Velikanov constant for a material consisting only of fine particles is the minimum permissible value of this parameter when transporting polydisperse material whose granulometric composition includes fine, small, and coarse fractions. It has also determined that the minimum permissible value of the M.A. Velikanov constant varies from 0 to 0.301.

For the first time, the dependence of the M.A. Velikanov constant on the parameters of the solid phase of the hydro-mixture and on the hydraulic friction coefficient in the critical hydrotransport regime has been established. This made it possible to derive formulas by which the CVHT is calculated using complexes of empirical constants.

It has been shown that as the CVHT increases, the value of the M.A. Velikanov constant also increases. At the same time, for engineering-accuracy calculations, the dependence of this constant on its minimum permissible value can be neglected. Based on the results of numerical studies, the possibility of approximating the dependence of the M.A. Velikanov constant on the turbulence influence parameter by power-law and logarithmic functions has been demonstrated.

Conflict of interest

Authors state no conflict of interest.

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ПІДВИЩЕННЯ ТОЧНОСТІ ВИЗНАЧЕННЯ КРИТИЧНОЇ ШВИДКОСТІ ГІДРОТРАНСПОРТУВАННЯ ПРИ ВИКОРИСТАННІ ГРАВІТАЦІЙНОЇ ТЕОРІЇ

Семененко Є.В., Блюсс Б.О., Хамініч О.В., Жанакова Р.К.

Анотація. Ефективність функціонування та енергоємність режимі роботи напірних гідротранспортних комплексів гірничих підприємств суттєво залежить від точності обчислення критичної швидкості гідротранспортування, що визначає мінімально припустиму швидкість руху гідросуміші, її можливу концентрацію та густину. Більшість формул, що використовуються для розрахунку критичної швидкості гідротранспортування

містять емпіричні константи, які змінюються в певних межах. Точних рекомендації по вибору величин цих констант не існує, що призводить до похибок при визначенні критичної швидкості гідротранспортування, а в цілому, або до завищення енергоємності процесу гідротранспортування, якщо це значення сильно завищено, або до нераціональних режимів роботи, якщо це значення занадто знижено. Найбільш чисельна група методів визначення критичної швидкості гідротранспортування спирається на гравітаційну теорію М.А. Великанова, та для визначення моменту реалізації критичного режиму використовує принцип пропорційності між додатковим гідравлічним ухилом та повним його значенням. Коефіцієнт цієї пропорційності є емпіричною величиною, та називається константою М.А. Великанова. Жодних рекомендації по визначенню цієї константи автор гравітаційної теорії течії гідросуміші не залишив, а науковці, що розробляли формули для розрахунку критичної швидкості гідротранспортування за таким принципом приховували її значення в інших експериментальних константах, які враховують вплив багатьох факторів. Зазначимо, що похибка, яка виникала при використанні цих формул, довгий час компенсувалася завищення критичної швидкості гідротранспортування. В економічних умовах колишнього СРСР, коли вартість електричної енергії була низькою, це не впливало на конкурентоспроможність вітчизняних підприємств. Але в нових економічних умовах, коли гірничо-видобувні підприємства країни конкурують на світовому ринку з більш розвинутими конкурентами, вимоги до енергоємності гідротранспортних комплексів підвищуються, що потребує більш точного визначення критичної швидкості гідротранспортування. В даній статті вперше визначається залежність коефіцієнтів в формулах для визначення критичної швидкості гідротранспортування від константи М.А. Великанова та пропонується методика визначення цього параметру в залежності від параметрів частинок твердої фази гідросуміші та критичної швидкості гідротранспортування.

Ключові слова: гідросуміш, гідротранспорт, критична швидкість, гравітаційна теорія.